Particle Emission from a Higher-Dimensional Black Hole

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Outline :

1. Extra Dimensions and Black Holes
2. Evaporation of Black Holes - Hawking Radiation
   - Schwarzschild Black Holes
   - Kerr Black Holes
3. Conclusions

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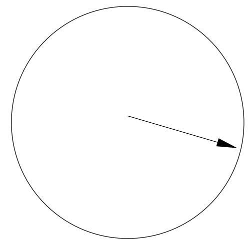
G. Duffy & M. Casals (Dublin)

Introduction : Extra Dimensions

- Kaluza & Klein (1921/1926): Apart from the usual 3 spacelike dimensions, additional, compact spacelike dimensions may exist in nature



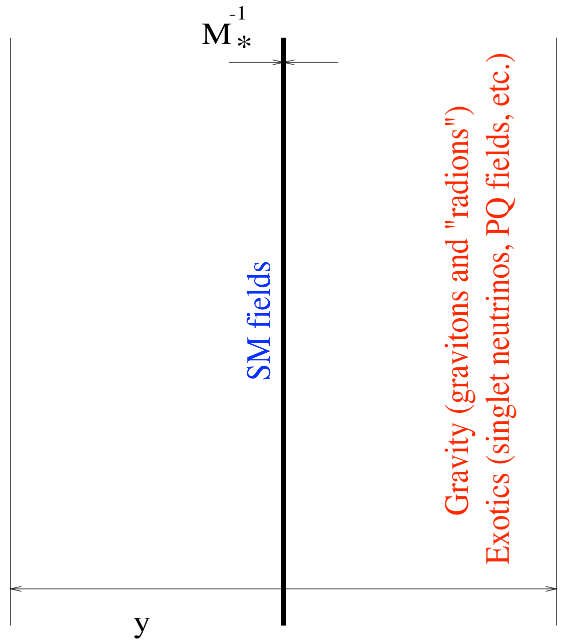
Why we can not observe them? Because of their tiny size $\mathcal{R}$: for experiments at $r \gg \mathcal{R}$, the extra dimensions are 'invisible'

- Superstring Theory: Formulated in $D = 10$ dims
  - (1975-90): $\mathcal{R} = l_P = 10^{-33}$ cm
  - (90's): $\mathcal{R} \geq (TeV)^{-1}$

    (Antoniadis; Lykken; Horava & Witten)

Introduction : Extra Dimensions

- Large Extra Dimensions (1998) :



  - A 4D **Brane** with all the SM fields and scale for gravity $M_P = 10^{19}$ GeV
  - A $(4+n)$D Extra Space (**Bulk**) with gravitons and scale for gravity M_*
  - Then, we obtain:

$$M_P^2 \simeq \mathcal{R}^n \, M_*^{2+n}$$

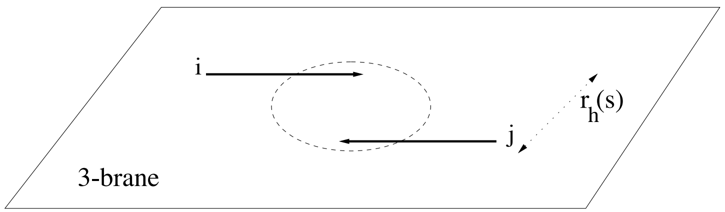
If $\mathcal{R} \leq 1$ mm, then $M_* \geq 1$ TeV $\Rightarrow$ resolution of the hierarchy problem (Arkani-Hamed, Dimopoulos & Dvali)

Introduction : Black Hole Creation

• During gravitational collapse on the brane, we expect the formation of a black hole, that will extend off the brane

• Small BH's with $r_H \ll \mathcal{R}$ are $(4+n)$-D objects

▷ Such Black Holes may be created during the scattering of high energy particles (Banks & Fischler; Giddings & Thomas; Dimopoulos & Landsberg)



For every center-of-mass-energy $\sqrt{s}$, there is a Schwarzschild radius r_H – if $b < r_H$, a black hole will be created (Thorne)

Evaporation of Black Holes

- Hawking Radiation: What is it?
  - creation of a virtual pair of particles just outside the horizon
  - the antiparticle falls into the BH whose mass decreases
  - the particle escapes to infinity where it gets observed

The Radiation Spectrum: A nearly black-body spectrum with emission rate

$$\frac{dE(\omega)}{dt} = \frac{|\mathcal{A}_n^{(s)}(\omega)|^2\ \omega}{\exp\left(\omega/T_{BH}\right) \mp 1}\ \frac{d\omega}{(2\pi)}$$

where $|\mathcal{A}_n^{(s)}(\omega)|^2$ is the absorption probability (greybody factor)

Schwarzschild Black Holes

- A spherically-symmetric $(4+n)$-dimensional BH with line-element (Tangherlini; Myers & Perry)

$$ds^2 = -\left[1 - \left(\frac{r_H}{r}\right)^{n+1}\right] dt^2 + \left[1 - \left(\frac{r_H}{r}\right)^{n+1}\right]^{-1} dr^2 + r^2 d\Omega^2_{2+n}$$

and temperature

$$T_H = \frac{n+1}{4\pi r_H}$$

To find the Absorption Probability, we must solve the corresponding Equation of Motion

- 4D Case : Teukolsky & Press; Starobinsky & Churilov; Unruh; Sanchez; Page; McGibbon & Webber

Schwarzschild Black Holes

- Emission on the brane: Scalar fields, fermions and gauge bosons living on our brane satisfy the e.o.m.

$$\Delta^s \frac{d}{dr}\left(\Delta^{1-s}\frac{dP_s}{dr}\right) + \left[\frac{\omega^2 r^2}{h} + 2i\omega s r - \frac{is\omega r^2 h'}{h} - \Lambda_{sj}\right] P_s(r) = 0$$

where

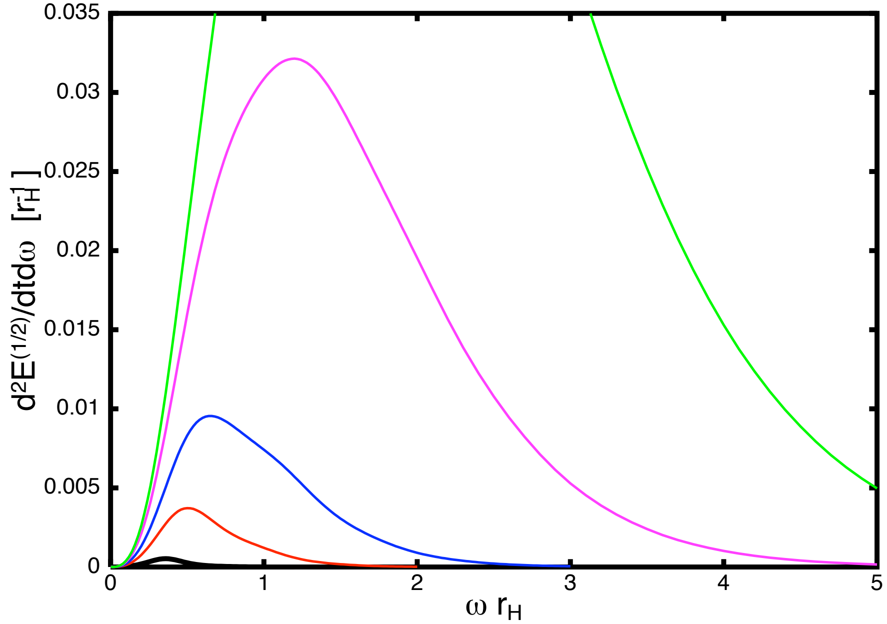
$$\Delta = r^2\, h = r^2 \left[1 - \left(\frac{r_H}{r}\right)^{n+1}\right], \qquad \Lambda_{sj} = j(j+1) - s(s-1)$$

$$\Psi_s = e^{-i\omega t}\, e^{im\varphi}\, \Delta^{-s}\, P_s(r)\, S^m_{s,j}(\theta)$$

This equation was solved analytically in the low-energy approximation (P.K. & March-Russell), and numerically (Harris & P.K.)

Schwarzschild Black Holes

• The amount of emitted energy strongly depends on the number of spacelike dimensions that exist transverse to the brane

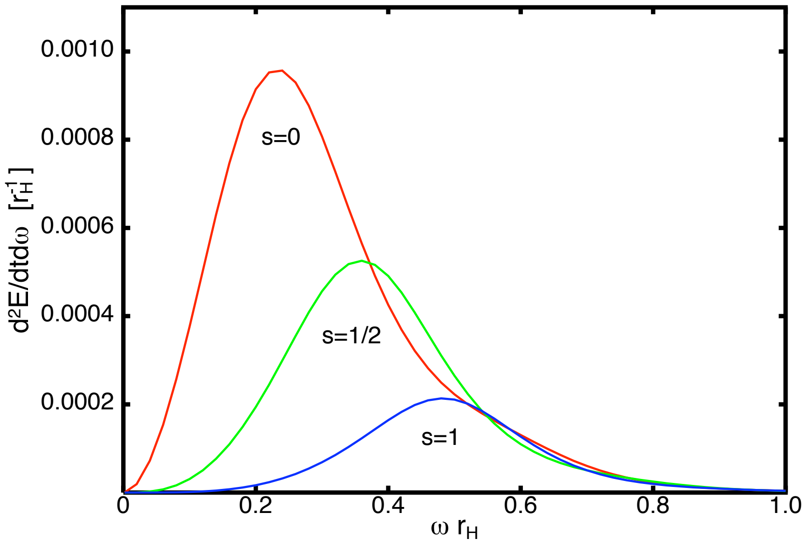


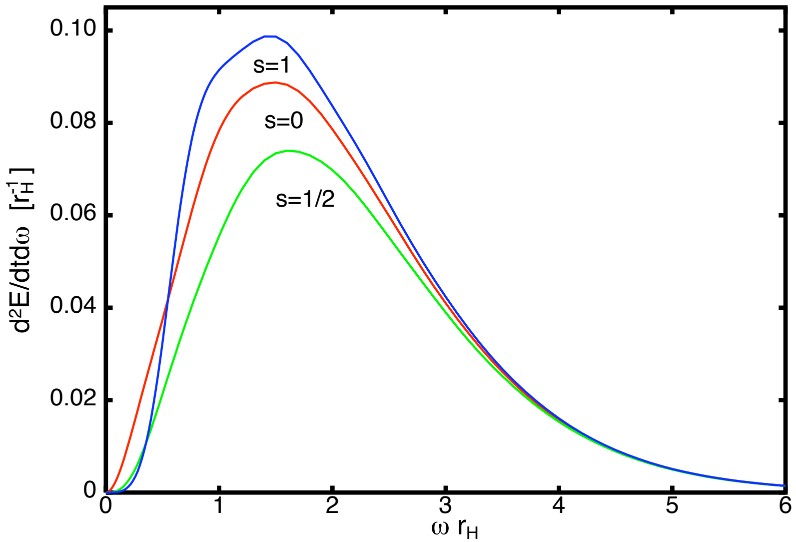
n	0	1	2	3	4	5	6	7
Scalars	1.0	8.94	36.0	99.8	222	429	749	1220
Fermions	1.0	14.2	59.5	162	352	664	1140	1830
G. Bosons	1.0	27.1	144	441	1020	2000	3530	5740

Schwarzschild Black Holes

- Relative Emission Rates : How do they change with n?

(Harris & P. K.)





$(n = 0)$ $\boxed{1 : 0.55 : 0.23}$ $(n = 6)$ $\boxed{1 : 0.84 : 1.06}$

- The type of the emitted radiation also depends strongly on n

Kerr Black Holes

- The line-element on the brane has the form of a rotating, neutral, n-dependent Myers-Perry solution

$$
\begin{aligned}
ds^2 &= \left(1-\frac{\mu}{\Sigma\, r^{n-1}}\right) dt^2 + \frac{2a\mu\sin^2\theta}{\Sigma\, r^{n-1}}\, dt\, d\varphi - \frac{\Sigma}{\Delta} dr^2 \\
&\quad -\Sigma\, d\theta^2 - \left(r^2 + a^2 + \frac{a^2\mu\sin^2\theta}{\Sigma\, r^{n-1}}\right) \sin^2\theta\, d\varphi^2,
\end{aligned}
$$

where

$$\Delta = r^2 + a^2 - \frac{\mu}{r^{n-1}} \quad \text{and} \quad \Sigma = r^2 + a^2\cos^2\theta$$

The parameters μ and a are associated to the black hole mass and angular momentum

Kerr Black Holes

- In this case, the "master" e.o.m. becomes (P.K.)

$$\Delta^{-s}\,\frac{d}{dr}\left(\Delta^{s+1}\,\frac{dR_s}{dr}\right)+$$

$$\left[\frac{K^2-iKs\Delta'}{\Delta}+4is\omega r+s\,(\Delta''-2)-\Lambda_\ell^m\right]R_s=0$$

where $K=(r^2+a^2)\,\omega-am$.

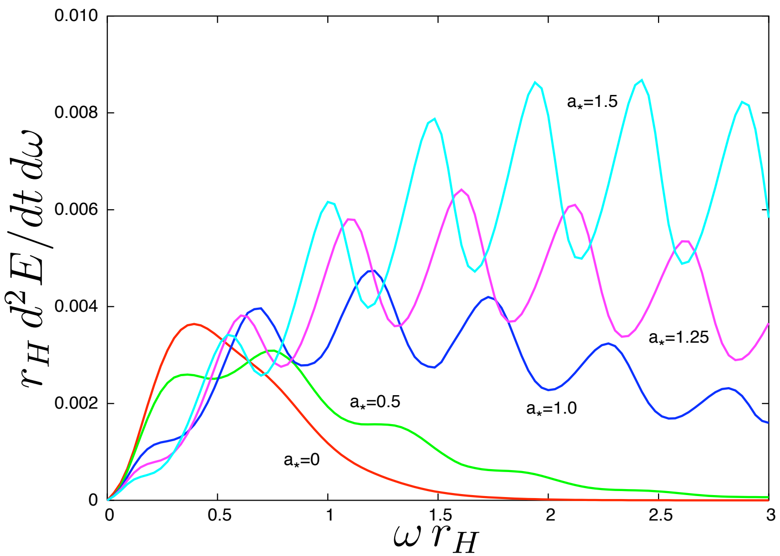
The Hawking temperature and rotation velocity of this brane black hole is

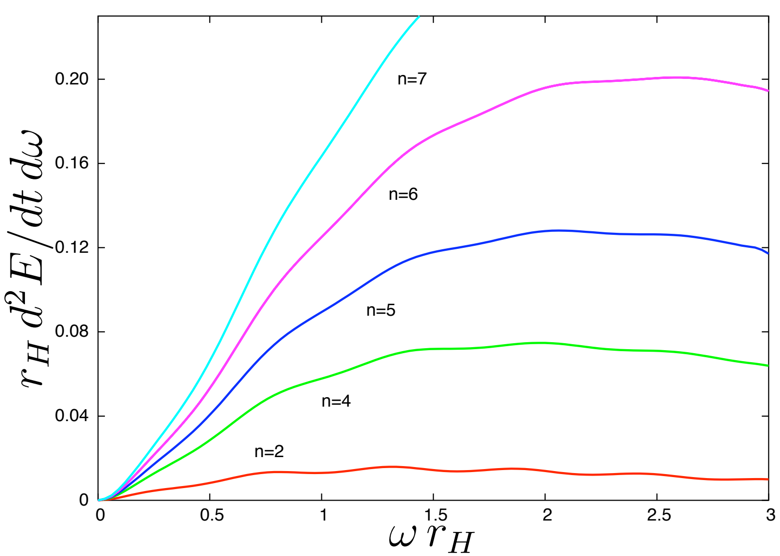
$$T_H=\frac{(n+1)\,r_H^2+(n-1)\,a^2}{4\pi(r_H^2+a^2)\,r_H}\,,\qquad \Omega=\frac{a}{(r_H^2+a^2)}$$

Kerr Black Holes

The differential energy emission rate is now given by:

$$\frac{dE(\omega)}{dt} = \sum_{\ell,m} |\mathcal{A}_{\ell,n}^m|^2 \, \frac{\omega}{\exp\left[(\omega - m\Omega)/T_H\right] - 1} \, \frac{d\omega}{2\pi}$$

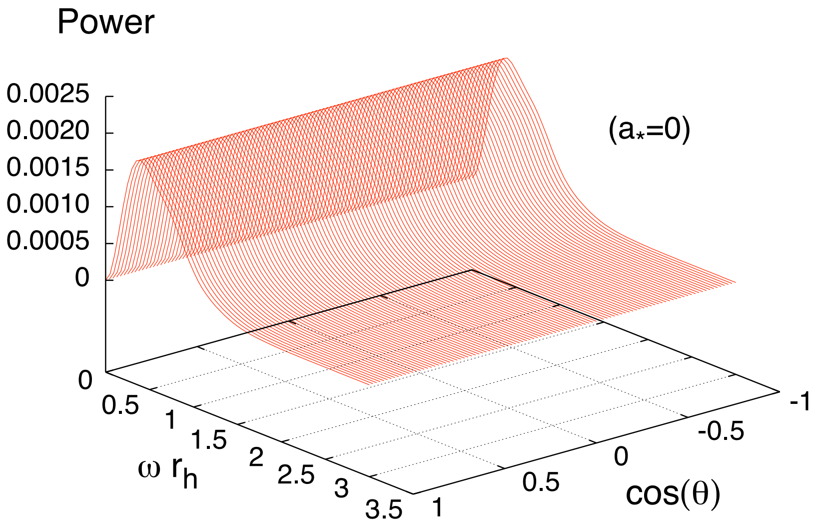


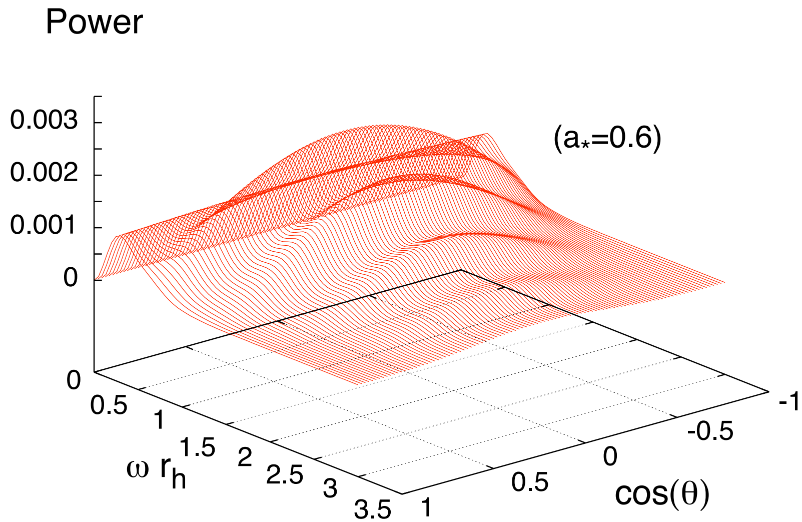


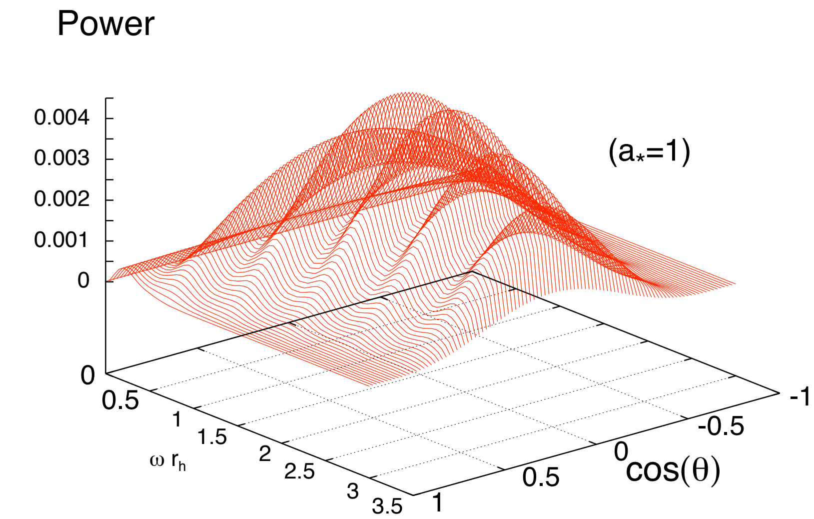
For scalar fields, as a or n increases, the energy emission rate is enhanced (Duffy, Harris, P.K. & Winstanley)

Kerr Black Holes

- Angular Distribution : For scalar fields, and $n = 1$, we find:





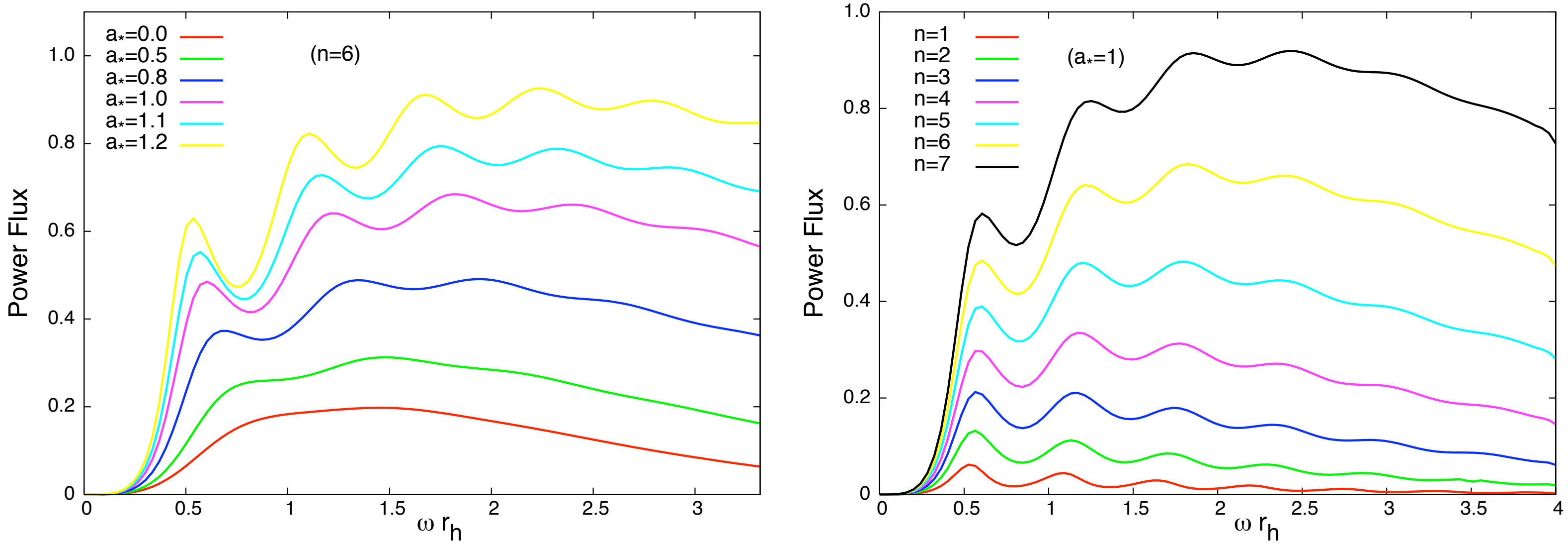


Emission for large a_* is mainly on a plane transverse to the axis of rotation (centrifugal potential)

Kerr Black Holes

For Gauge Bosons, a similar behaviour was found:

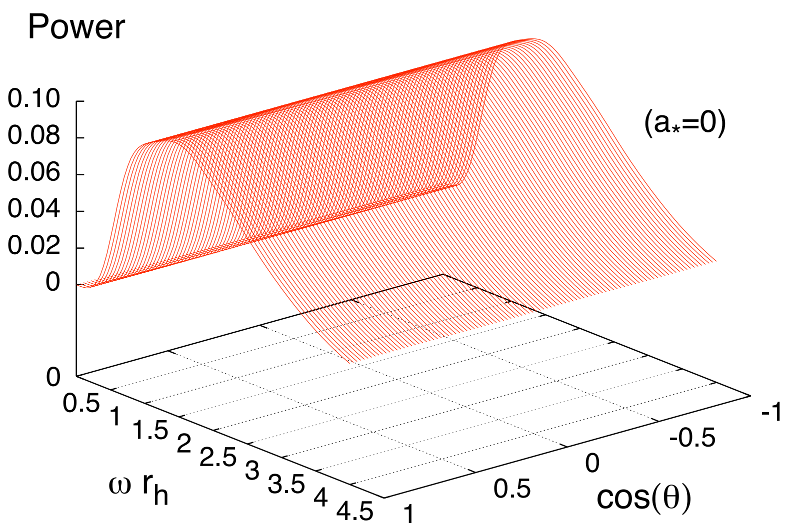
(Casals, P.K. & Winstanley)

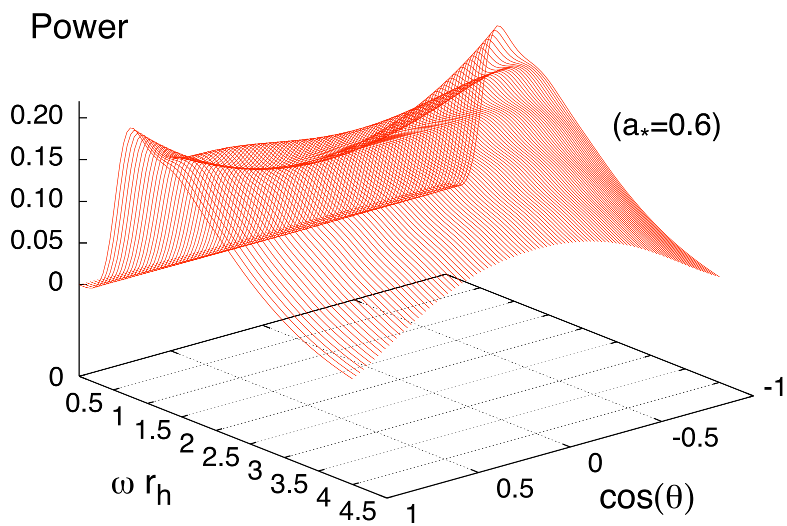


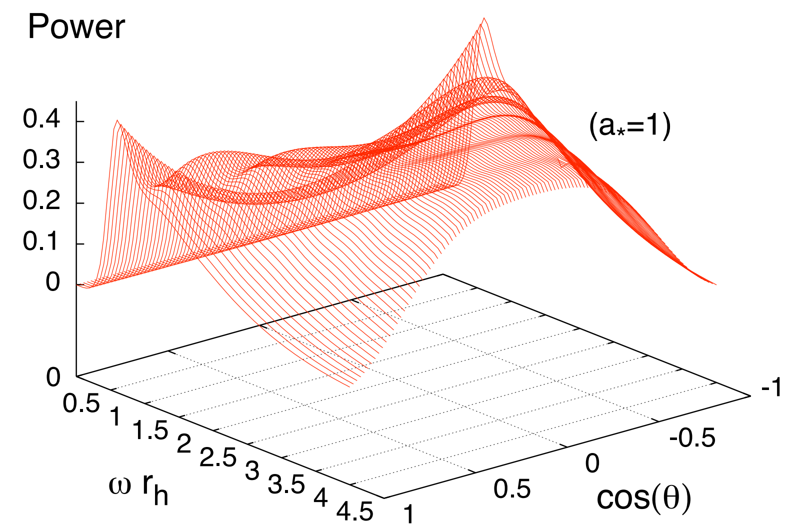
Again, as a or n increases, the energy emission rate is enhanced at all energies, particularly at the high-energy regime

Kerr Black Holes

- Angular Distribution : For gauge bosons, and $n = 6$, we find:







Small a_*: emission is mainly parallel to the axis of rotation (spin-rotation coupling) Large a_*: the centrifugal potential takes over again

Kerr BH's: Relative Emission Rates

- Gauge Bosons vs. Scalars : A rotating black hole prefers to emit gauge bosons instead of scalar fields
  - For $n = 1$, the emission rate for gauge bosons is almost an order of magnitude larger than the one for scalars
  - For $n = 4$, the emission rate for gauge bosons is almost 7 times larger than the one for scalars

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- Angular Momentum vs. Mass :
  - For large a, the angular momentum emission rate is significantly larger than the mass emission rate
  - For small a, the angular momentum emission rate falls well below the one for the mass

Conclusions

- Until today, we have not yet detected the Hawking radiation from a decaying black hole
- If Extra Dimensions exist, then
  - the production of small black holes becomes possible
  - the detection of Hawking radiation becomes more likely
- The emission spectra can help us determine the dimensionality of spacetime with possible signatures being the rate and type of the emitted radiation
- Both the Schwarzschild and the spin-down phases are now under intense investigation, and they both offer distinct signatures in their emission spectra