MASSIVE NEUTRINOS: MODEL BUILDING

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- Neutrino data:

By now convincing for $m_\nu \neq 0$ and physics beyond SM

- What do we know?

	lower limit	best value	upper limit
$\Delta m^2_{sun}(10^{-5}\ eV^2)$	5.4	6.9	9.5
$\Delta m^2_{atmo}(10^{-3}\ eV^2)$	1.4	2.6	3.57
$\sin^2\theta_{12}$	0.23	0.30	0.39
$\sin^2\theta_{12}$	0.31	0.52	0.72
$\sin^2\theta_{13}$	0	0.006	0.1

- Open questions:

- Is the atmospheric mixing maximal or close to maximal?
- How large is the solar mixing?
- What about θ_{13} ?
- What is the pattern of masses? (*we know only* Δm)
- Phases? How large is leptonic CP-violation?

- Origin of neutrino mass in SM extensions
  - Family symmetries (abelian? non-abelian?)
  - GUTs (which one?) / SUSY?

  Use (i) multiplet structure

  (ii) known fermion mass and mixing parameters

  to predict those we know less

- Additional aspects to consider:
  - Stability under quantum corrections (large effects possible)
  - Leptonic CP-violation/Baryogenesis through leptogenesis?
  - Rare charged-lepton decays, μ-e conversion on nuclei
  - Collider Signatures/LHC

- Combined analysis of the above for max. information

$3\times\ 3$ mixing

$$|\nu_a >= \sum_i U_{ai}|\nu_i >, \quad a = e, \mu, \tau; \;\; i = 1, 2, 3$$

$$U = diag(e^{i\delta_e}, e^{i\delta_\mu}, e^{i\delta_\tau}).V.diag(e^{-i\phi_1/2}, e^{-i\phi_2/2}, 1)$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$\delta = -\arg\left(\frac{\frac{U_{ii}^* U_{ij} U_{ji} U_{jj}^*}{c_{12}c_{13}^2 c_{23}s_{13}} + c_{12}c_{23}s_{13}}{s_{12}s_{23}}\right), i, j = 1, 2, 3 \text{ and } i \neq j$$

i.e. CP-violation $\propto$ Jarlskog invariant

$$\begin{aligned} J_{CP} &= \frac{1}{2}|Im(U_{11}^* U_{12} U_{21} U_{22}^*)| = \frac{1}{2}|Im(U_{11}^* U_{13} U_{31} U_{33}^*)| \\ &= \frac{1}{2}|Im(U_{22}^* U_{23} U_{32} U_{33}^*)| = \frac{1}{2}|c_{12}c_{13}^2 c_{23} \sin\delta s_{12}s_{13}s_{23}| \end{aligned}$$

Different models predict different CP violation

i.e. zero violation, for models with texture zeros in (1,3) entries

ν_R and See-Saw mechanism

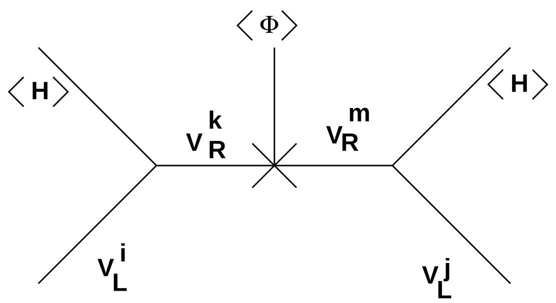
How can we generate naturally light neutrinos?

Combine m_ν^D and M_{ν_R} to write a mass matrix

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_\nu^D \\ m_\nu^D & M_{\nu_R} \end{pmatrix}$$

If $M_{\nu_R} \gg m_\nu^D$, a very heavy eigenvalue $M_N \approx M_{\nu_R}$ and a very light

$$m_{eff} \approx \left| \frac{(m_\nu^D)^2}{M_{\nu_R}} \right|$$



For $(m_\nu^D)_{33} \approx (200\,\text{GeV})$ $(\lambda_N \approx \lambda_t)$ and $M_{N_3} \approx \mathcal{O}(10^{13}\,\text{GeV})$,

$m_{eff} \approx 1$ eV

Mass hierarchies and flavour symmetries

- Start with L-R symmetric model, assume flavour symmetry

(different generations of fermions have different charges)

Invariance under symmetry, determines magnitude of masses

	Q_i	$\bar{U}_i$	$\bar{D}_i$	L_i	$\bar{E}_i$	H_2	H_1
$U(1)$	a_i	a_i	a_i	b_i	b_i	$-2a_3$	wa_3

- LR + $SU(2)$ $\Rightarrow$ same charge for $Q_i, \bar{U}_i, \bar{D}_i$
- Up-mass matrix:

  Top coupling $Q_3\bar{U}_3H_2$ 0 charge $\Rightarrow$ allowed

  All other couplings forbidden

$$M^{up} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Suppose singlets θ with non-0 flavor-charges

  (singlets expected in realistic models)

  Then: invariant terms $Q_i\bar{U}_jH_2(<\theta>/M)^n$

  n depending on flavour charges
- Hierarchical mass structures generated for ALL fermions

Example consistent with charged fermion hierarchies

$$M^u \propto \begin{pmatrix} \epsilon^8 & \epsilon^3 & \epsilon^4 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^4 & \epsilon & 1 \end{pmatrix}, M^d \propto \begin{pmatrix} \bar\epsilon^8 & \bar\epsilon^3 & \bar\epsilon^4 \\ \bar\epsilon^3 & \bar\epsilon^2 & \bar\epsilon \\ \bar\epsilon^4 & \bar\epsilon & 1 \end{pmatrix}$$

$$M_\ell \propto \begin{pmatrix} \bar\epsilon^5 & \bar\epsilon^3 & \bar\epsilon^{5/2} \\ \bar\epsilon^3 & \bar\epsilon & \bar\epsilon^{1/2} \\ \bar\epsilon^{5/2} & \bar\epsilon^{1/2} & 1 \end{pmatrix}, V_{tot} = V_\nu^\dagger V_\ell$$

- If mixing mostly dominated by V_ℓ:

- naturally large neutrino mass hierarchies

- simplest constructions with too small solar mixing

(fixed by charged lepton hierarchies)

- If R-H neutrino sector relevant for mixing

see-saw 0-determinant cancellations, large solar mixing

- Models with more than one U(1)'s and more than one singlets

-less predictive, but often simulate well non-abelian structures

(very instructive to study them).

Minimal Models with Abelian Flavour Symmetries

- Large splitting between fermion masses

  Naturally leads to large neutrino hierarchies

- Unknown phases/order unity coefficients $\Downarrow$

  Difficult to obtain naturally degenerate neutrinos

- In many models lepton hierarchies consistent with mostly SAMSW but LAMSW possible, ie by see-saw conditions

Models with non-Abelian flavour symmetries

- Degenerate ν and $\ell^{\pm}$ textures assuming

  ie that the lepton fields are SO(3) triplets

- Subsequently break SO(3) so as:

  large charged lepton splitting/ small neutrino splitting

- Favour almost-degenerate neutrino textures

- Textures with (almost)-bimaximal mixing predicted

  LAMSW / VO oscillations for solar neutrinos

$SU(5)$

(i) Assume the family symmetry is combined with $SU(5)$

(ii) Use the GUT structure ONLY to constrain $U(1)$ charges

Under this group we have the following relations:

$$\begin{aligned} Q_{(q,u^c,e^c)_i} &= Q_i^{10} \\ Q_{(l,d^c)_i} &= Q_i^{\bar{5}} \\ Q_{(\nu_R)_i} &= Q_i^{\nu_R} \end{aligned}$$

- M_{up} symmetric
- $M_{\ell^\pm} = M_{down}^T$
- L lepton mixing $\approx$ R down-quark one

Can we obtain acceptable patterns of masses/mixings? i.e.

$$\frac{M_u}{m_t} = \begin{pmatrix} \bar{\epsilon}^6 & \bar{\epsilon}^5 & \bar{\epsilon}^3 \\ \bar{\epsilon}^5 & \bar{\epsilon}^4 & \bar{\epsilon}^2 \\ \bar{\epsilon}^3 & \bar{\epsilon}^2 & 1 \end{pmatrix}, \quad \frac{M_{down}}{m_b} = \begin{pmatrix} \bar{\epsilon}^4 & \bar{\epsilon}^3 & \bar{\epsilon}^3 \\ \bar{\epsilon}^3 & \bar{\epsilon}^2 & \bar{\epsilon}^2 \\ \bar{\epsilon} & 1 & 1 \end{pmatrix}$$

$$\frac{M_\ell}{m_\tau} = \begin{pmatrix} \bar{\epsilon}^4 & \bar{\epsilon}^3 & \bar{\epsilon} \\ \bar{\epsilon}^3 & \bar{\epsilon}^2 & 1 \\ \bar{\epsilon}^3 & \bar{\epsilon}^2 & 1 \end{pmatrix}$$

Solar Mixing: more structure needs to be added!

Ellis, Gomez, ML: computerised scanning of viable constructions, in progress

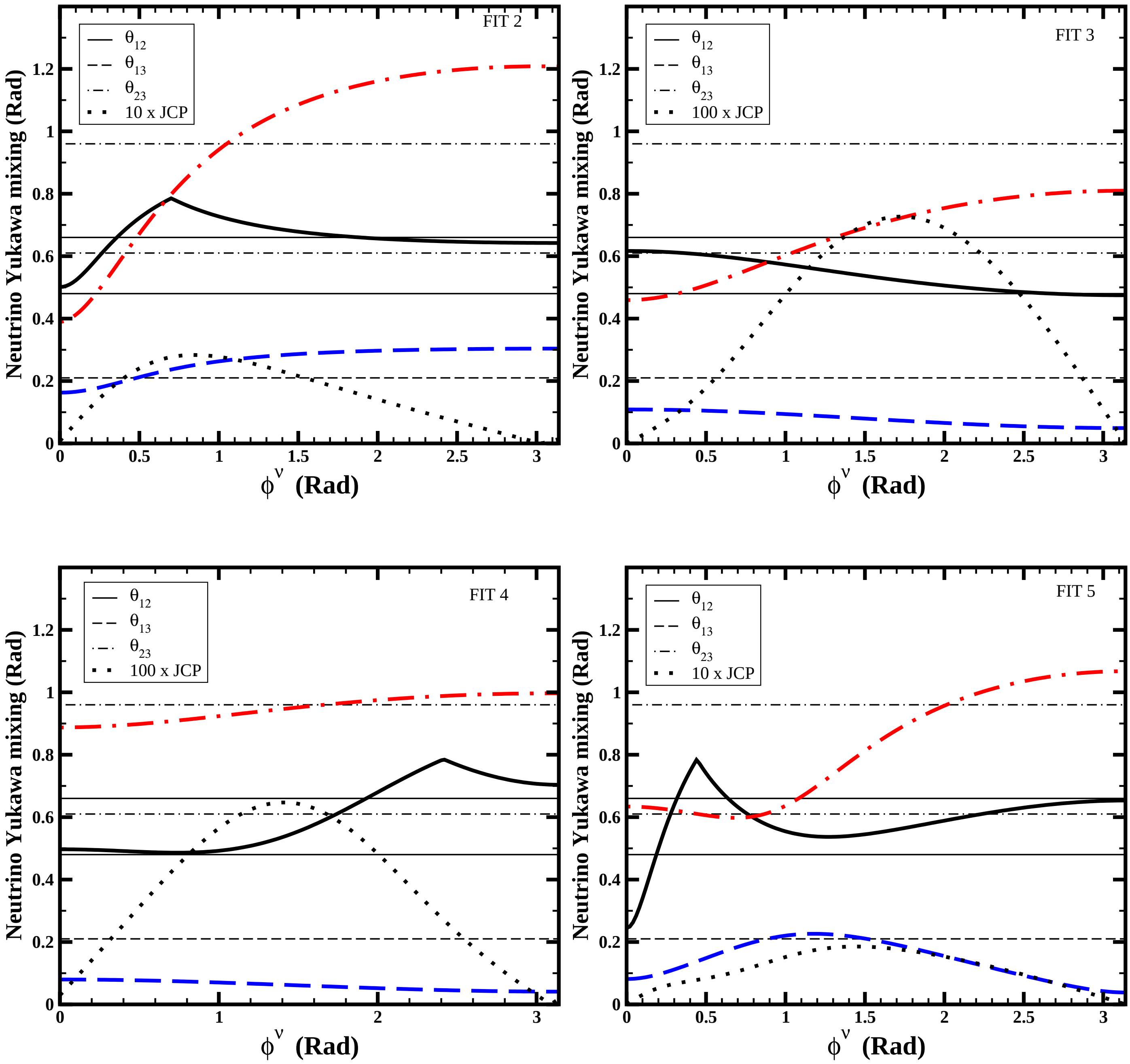


Figure 1: *Neutrino mixing angles and values J_{CP} vs. the phase ϕ_ν of the Dirac neutrino matrix Y_ν. The phase $\phi_{X_{23}}$ of the Y_e is kept constant (and the results are almost independent of it). The texures corresponds to fits 2,3,4,5 of KKPV with the values of coef. a_{ij} provided in their tables 11 and 13.*

$SO(10)$

- All L- and R-handed fermions in the 16 of $SO(10)$
- Both MSSM Higgs fields fit in a single 10 of $SO(10)$ $\Downarrow$

For all fermions, *L-R symmetric textures*, similar structure (different expansion parameters due to Higgs mixing)

Flipped $SU(5)$

$$Q_{(q,d^c,\nu^c)_i} = Q_i^{10}, \quad Q_{(l,u^c)_i} = Q_i^{\bar{5}}, \quad e^c \text{ singlet of } SU(5)$$

- Symmetric M_{down}
- $m_\nu^D = M_{up}^T$

$SU(3)_c \otimes SU(3)_L \otimes SU(3)_R$

Particles placed in $(3,3,1)$, $(\bar{3},1,\bar{3})$ and $(1,3,\bar{3})$ as:

$$\begin{pmatrix} u \\ d \\ D \end{pmatrix}_L \quad \begin{pmatrix} \bar{u} & \bar{d} & \bar{D} \end{pmatrix}_L \quad \begin{pmatrix} \ell^c & L & e^- \\ L^c & \ell & \nu \\ e^+ & \nu^c & N \end{pmatrix}_L$$

- Symmetric lepton mass matrices (as in L-R symm. models)
- Asymmetric up and down

Different predictions and correlations between observables

Baryogenesis through leptogenesis

- Neutrinos have masses and mix with each other
- Like quarks, CP violation in neutrino sector

$$\mathcal{L} = \overline{\ell_L}\, \Phi\, h_\nu\, N_L^c + \frac{1}{2}\overline{N_L^c}\, M\, N_L^c + \text{h.c.}$$

- Lepton-number-violation

(i.e. in decays of heavy, RH Majorana neutrinos)

$$N_L^c \;\rightarrow\; \overline{\Phi} + \ell$$

$$N_L^c \;\rightarrow\; \Phi + \overline{\ell}$$

REMEMBER:

L/B-violating interactions in thermal equilibr. at high T

Changes in lepton number $\Rightarrow$ Changes in baryon number

THUS:

Generate $\Delta L \neq 0$, which then transforms to $\Delta B \neq 0$

Out-of-equilibrium condition:

Decay rates smaller than Hubble parameter H at $T \approx M_{N_1}$

Three-level width of N_1: $\Gamma = \frac{(\lambda^\dagger\lambda)_{11}}{8\pi} M_{N_1}$

Compare with: $H \approx 1.7\, g_*^{1/2}\, \frac{T^2}{M_p}$

$(g_*^{MSSM} \approx 228.75,\ g_*^{SM} = 106.75)$

$$\Rightarrow \frac{(\lambda^\dagger\lambda)_{11}}{14\pi g_*^{1/2}} M_p < M_{N_1}$$

More accurate by looking at Boltzmann equations

CP-violating asymmetry, ϵ

(interference between tree-level and 1-loop amplitudes)

$$\epsilon_j \ = \ \frac{1}{(8\pi\lambda^\dagger\lambda)_{11}} \sum_j \mathrm{Im}\left[(\lambda^\dagger\lambda)^2_{1j}\right] f\left(\frac{m^2_{N_j}}{m^2_{N_1}}\right)$$

$$f(y) \ = \ \sqrt{y}\left[1 - (1+y)\ln\left(\frac{1+y}{y}\right)\right]$$

Plus self-energy corrections $\tilde{\delta} \propto \frac{M_{N_1}}{(M_{N_2} - M_{N_1})}$

What can leptogenesis tell us

about fermion mass patterns?

Effects of radiative corrections on neutrino masses and mixing

For i, j, generation indices

$$\frac{1}{m^{ij}_{eff}}\frac{d}{dt}m^{ij}_{eff} = \frac{1}{8\pi^2}\left(-c_i g_i^2 + 3\lambda_t^2 + \frac{1}{2}(\lambda_i^2+\lambda_j^2)\right)$$

$$16\pi^2\frac{d}{dt}\sin^2 2\theta_{23} = 2\sin^2 2\theta_{23}(1-2\sin^2\theta_{23})\lambda_\tau^2\frac{m^{33}_{eff}+m^{22}_{eff}}{m^{33}_{eff}-m^{22}_{eff}}$$

$\sin^2 2\theta_{23}$ affected by quantum corrections if:

(i) λ_τ large (large $\tan\beta$) (ii) $m^{33}_{eff} - m^{22}_{eff}$ small

Semi-analytic and numerical studies $\Rightarrow$

- The mixing can even be amplified/destroyed

$$\frac{m^{ij}_{eff}}{m^{ij}_{eff,0}} = exp\left\{\frac{1}{8\pi^2}\int_{t_0}^{t}\left(-c_i g_i^2 + 3\lambda_t^2 + \frac{1}{2}(\lambda_i^2+\lambda_j^2)\right)\right\}$$
$$\equiv I_g \cdot I_t \cdot \sqrt{I_i} \cdot \sqrt{I_j}$$

1. The relative structure in m_{eff} is only modified by the leptonic Yukawa couplings

2. On the contrary, the gauge and top couplings give only an overall scaling factor

LFV IN RARE DECAYS AND CONVERSIONS

In SM extensions with $\Delta L_i \neq 0$, non-zero rates for processes such as:

$$\mu \rightarrow e\gamma$$

$$\tau \rightarrow \mu\gamma$$

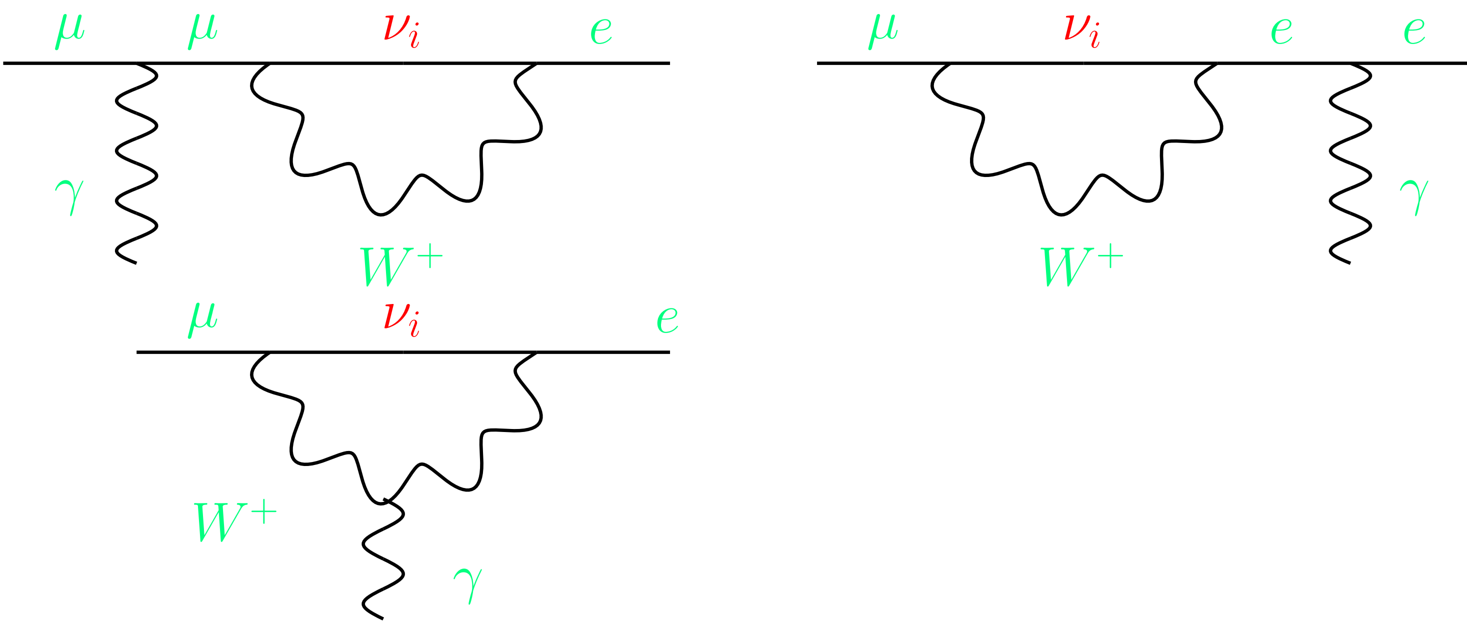
$$\mu - e \ \text{conversion on nuclei}$$

Very good expected future BR sensitivities:

$$\mu \rightarrow e\gamma \qquad 10^{-14}$$

$$\mu^- Ti \rightarrow e^- Ti \quad 10^{-18}$$

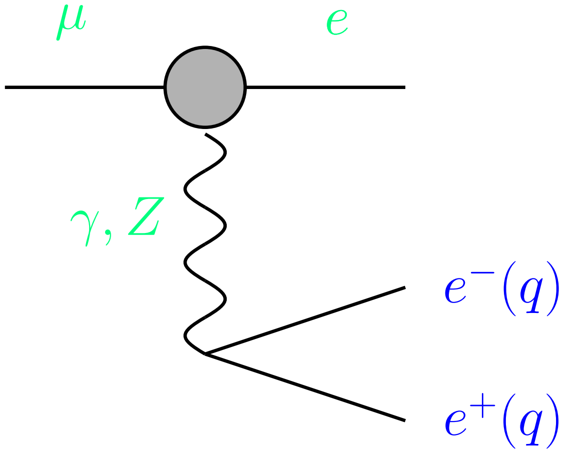
1. $\mu \to e\gamma$ in the SM with $m_{\nu_i} \neq 0$



$\nu_i = \nu_\mu \cos\theta + \nu_e \sin\theta$, $\Gamma = \frac{1}{16}\frac{G_F^2\ m_\mu^5\ \alpha}{128\ \pi^4}\left(\frac{m_2^2 - m_1^2}{m_W^2}\right)\sin^2\theta\cos^2\theta$

$BR \leq 10^{-50}$, for Δm_{12}^2 from neutrino data too small!

2. $\mu \to 3e$ et μ -e conversion on nuclei



+ ...(suppressed box-diagr.)

$BR \leq 10^{-53}$

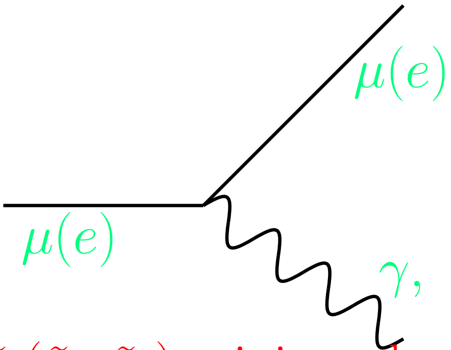
for Δm_{12}^2 from neutrino data

LFV in minimal SUSY

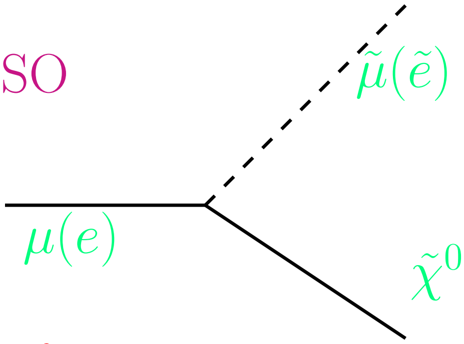
MSSM: For each SM vertex, also the one with

2 particles $\rightarrow$ superparticles

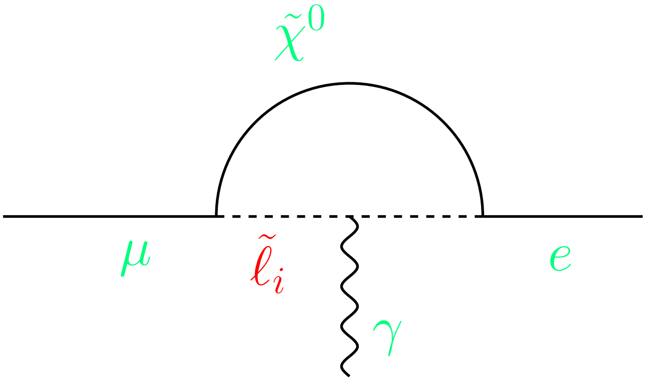
FOR

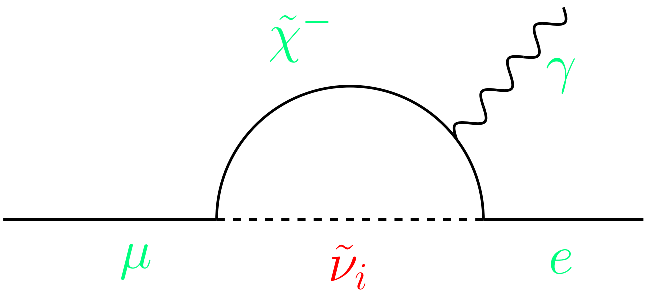


ALSO



If $\tilde{\mu}$-$\tilde{e}$ ($\tilde{\nu}_\mu$-$\tilde{\nu}_e$) mixing, large rates for:





The fermion in the loop is now a neutralino/chargino instead of a neutrino

($m_{\tilde{\chi}^0}, m_{\tilde{\chi}^\pm} \gg m_\nu \Rightarrow$ large rates)

The magnitude of the rates depends on:

The mass of superparticles

The mixing of superparticles

For non-universality at $m_{\rm GUT}$, large rates

Massive neutrinos and SUSY

Even if:

$$M_{\rm GUT} : m_{\tilde{\ell},\tilde{\nu}} \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \text{REGs} \longrightarrow \begin{pmatrix} 1 & \star & \star \\ \star & 1 & \star \\ \star & \star & 1 \end{pmatrix}$$

- RGEs for the charged-lepton mass matrix

$$t\frac{d}{dt}(m^2_{\tilde{\ell}})^j_i = \frac{1}{16\pi^2}\left\{(m^2_{\tilde{\ell}}\lambda^\dagger_\ell\lambda_\ell)^j_i + (m^2_{\tilde{\ell}}\lambda^\dagger_\nu\lambda_\nu)^j_i + \ldots\right\}$$

The corrections in the basis where $(\lambda^\dagger_\ell\lambda_\ell)^j_i$ is diagonal, are:

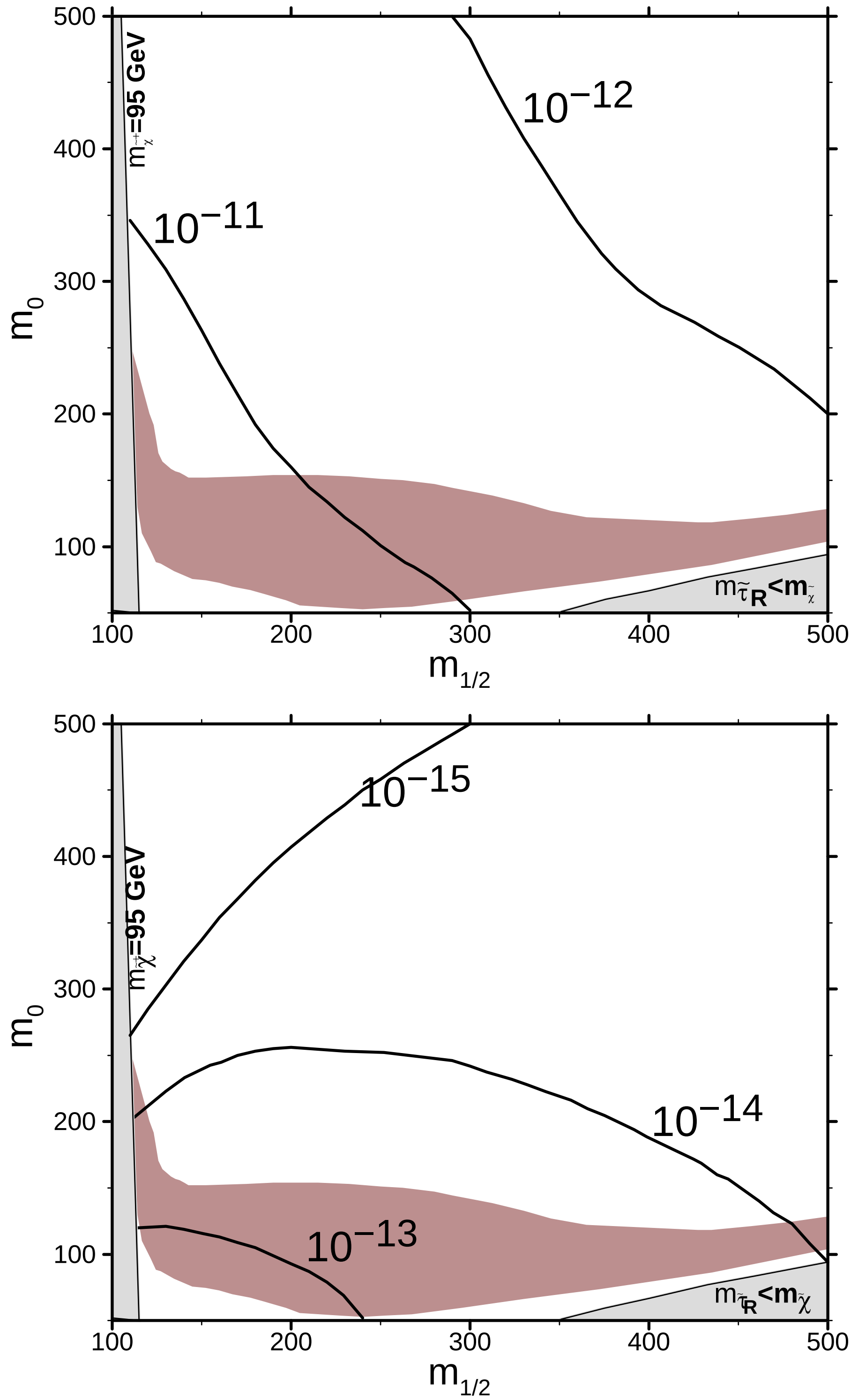
$$\delta m_{\tilde{\ell}} \propto \frac{1}{16\pi}\ln\frac{M_{\rm GUT}}{M_N}\lambda^\dagger_\nu\lambda_\nu m^2_{\rm SUSY}$$

(And similar corrections for $\delta m_{\tilde{\nu}}$)

INFO: For big μ-e lepton mixing, big rates for $\mu \to e\gamma$

Different predictions for the various solutions of the solar neutrino deficit (with a small/large mixing angle and with eV or $\approx$ 0.01 eV neutrinos)

If R-violation, tree-level $\mu \to 3e$ *and* $\mu - e$ *conversion!*



Predictions for $\mu \to e\gamma$ and $\mu - e$ conversion for the (by now excluded) "SAMSW" and $m_{\nu_3} \gg m_{\nu_{1,2}}$.
For large solar mixing, naturally, even larger rates are predicted for the same set of parameters.

LFV at LHC:

Most promising: LFV decay of $\tilde{\chi}_2^0$

$$\tilde{\chi}_2^0 \to \tilde{\ell}_i^+ \ell_j^- \to \tilde{\chi}_1^0 \ell_i^+ \ell_j^-$$

Background:

$$\tilde{\chi}_2^0 \to \chi_1^0 Z(h) \to \tilde{\chi}_1^0 \ell_i^+ \ell_i^-$$

(*$\tilde{\chi}_2^0$ produced through $\tilde{q}, \tilde{g}$*) decays

-60% of 1st- and 2nd-generation LH-squarks decay in Wino-like neutralino and chargino

- RH-slepton masses expected to be smaller than $m_{\tilde{\chi}_2^0}$

- If $\mu \gg M$, most parameter space excluded by rare muon and tau decays.
- If μ relatively close to gaugino masses,

LHC covers range where rare decays suppressed

(cancellations between chargino and neutralino diagrams)

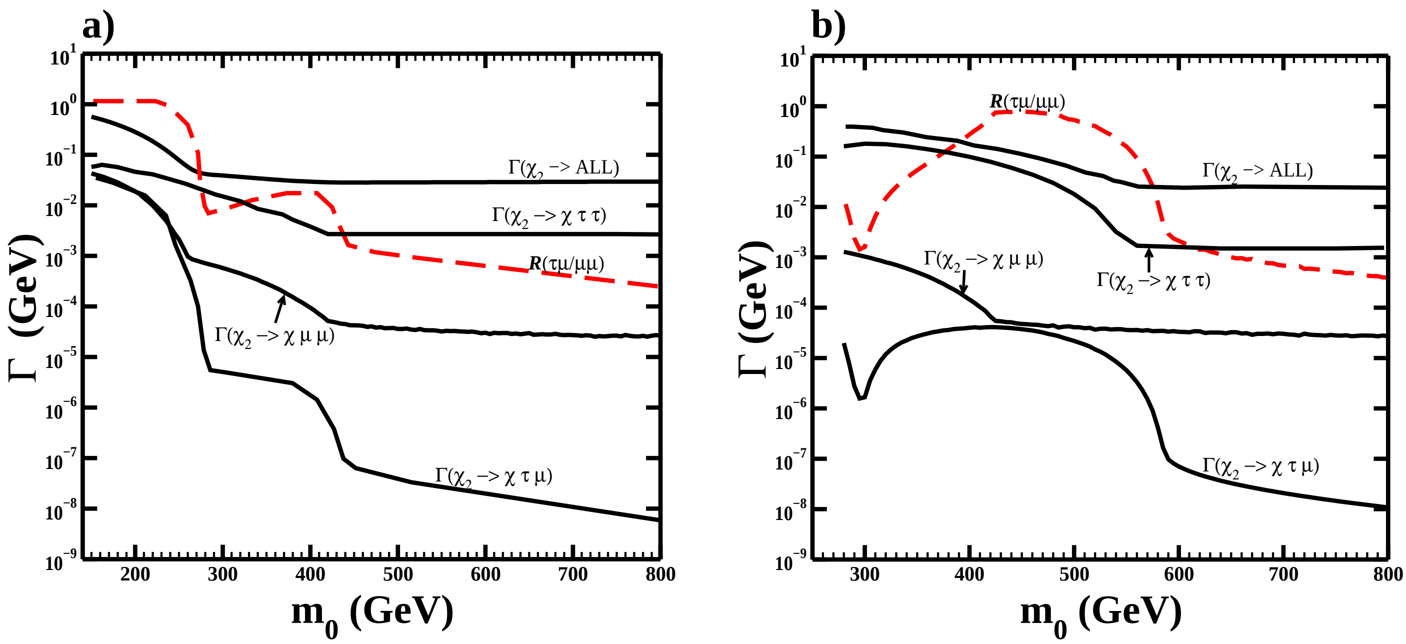


Figure 2: *Comparison of flavour-changing and -conserving* χ_2 *decay modes as functions of* m_0 *for (a)* $\tan\beta = 10, \mu > 0, m_{1/2} = 600$ *GeV and (b)* $\tan\beta = 40, \mu > 0, m_{1/2} = 600$ *GeV. We assume for illustration a non-universality factor* $x = 0.9$ *and a mixing angle* $\phi = \frac{\pi}{6}$.

Carvahlo, Ellis, Gomez, ML, Romao, PLB2005

CONCLUSIONS

♦ Neutrino Oscillations

Neutrino masses / $\Delta L \neq 0 \Downarrow$

Extensions of SM (L-R, GUTS, $\not{R}_p$, ...)

♦ Phenomenological Textures

Which solutions?

Which correlations between different parameters?

Further predictions?

♦ Implications for underlying theory

Different models "prefer" different solutions.

New data progressively helps us to exclude/constrain the models

In Patras, EXT EC grant

for research and collaborations on these areas

Available funding for:

- A PhD post
- A Post-doc
- Several research visits (2-3 months each)

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http://www.physics.upatras.gr/ magda/group/EXT.htm