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Fluxes - Gaugings and Superpotentials in Superstring Theories

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1. Introduction

• Compactifications of Superstrings and M-theory provide a plethora of 4d- vacua with exact or (spontaneously) broken supersymmetries.

• Phenomenologically interesting are those with Chiral Fermions

$$N = 8, 4 \qquad \longrightarrow \qquad N = 1 \rightarrow N = 0$$

• The underlying D = 10 theories encode $N \geq 4$ constrained structure which can be used to obtain useful information on the effective $N = 1$ supergravity.

• The 4d $N = 1$ theories, typically include moduli fields whose vacuum expectation values are undetermined.

Some of these moduli are the: dilaton field Φ, internal metric fields G_{IJ}, and p-form fields F^p

Generating a potential for some of the moduli is essential in order to :

- **reduce the number of massless scalars**
- **induce supersymmetry breaking**
- **determine the (3+1)d geometry**

In the $N \geq 4$ supergravity theories, the only available tool for generating a non-trivial potential is the "gauging".

"Gauging" $\rightarrow$ We introduce in the theory a gauge group G acting on the vector fields of the gravitational and the vector super-multiplets.

The important fact is: The kinetic terms of the fields in the gauged theory, remain the same as in the ungauged theory.

In the language of $N = 1$ $(\longleftarrow N \geq 4)$

The gauging modifications are non-trivial for the the superpotential W.

The Kähler potential K remains the same as in the ungauged theory.

To be more precise consider the case of superstring constructions with an $N = 4$ **supersymmetry:**

- **Heterotic on** T^6
- **Type IIA or IIB on** $K3 \times T^2$
- **Type IIA, IIB on orientifolds**
- **Type IIA, IIB asymmetric (4,0)**
- ...

2. $N=4$ Gauging $\leftrightarrow$ $N=1$ Superpotential

Independently of our starting point, the scalar manifold M of the induced $N=4$ effective supergravity is identical for all superstring constructions.

$$M = \left(\frac{SU(1,1)}{U(1)}\right)_S \times \left(\frac{SO(6,6+n)}{SO(6)\times SO(6+n)}\right)_{T_A,U_A,Z_I}$$

After $Z^2 \times Z^2$ orbifold (CY) projections

$N=4 \rightarrow N=1$ **and** $M \rightarrow K$

$$K = \left(\frac{SU(1,1)}{U(1)}\right)_S \times \left(\frac{SO(2,2+n_1)}{SO(2)\times SO(2+n_1)}\right)_{T_1,U_1,Z_1^I} \times \left(\frac{SO(2,2+n_2)}{SO(2)\times SO(2+n_2)}\right)_{T_2,U_2,Z_2^I} \times \left(\frac{SO(2,2+n_3)}{SO(2)\times SO(2+n_3)}\right)_{T_3,U_3,Z_3^I}$$

$$K = -\log\left(S+\bar{S}\right)$$

$$-\log\left((T_1+\bar{T}_1)(U_1+\bar{U}_1)-(Z_1+\bar{Z}_1)^2\right)$$

$$-\log\left((T_2+\bar{T}_2)(U_2+\bar{U}_2)-(Z_2+\bar{Z}_2)^2\right)$$

$$-\log\left((T_3+\bar{T}_3)(U_3+\bar{U}_3)-(Z_3+\bar{Z}_3)^2\right).$$

The above choice of parameterization is a solution to the $N=4$ constraints after $Z^2\times Z^2$ orbifold projections $N=4\rightarrow N=1$:

S-manifold

$$|\phi_0|^2-|\phi_1|^2=\frac{1}{2}\qquad\longrightarrow$$

$$\phi_0-\phi_1=\frac{1}{(S+\bar{S})^{1/2}},\qquad\qquad \phi_0+\phi_1=\frac{S}{(S+\bar{S})^{1/2}}$$

T_A, U_A, Z^I_A-manifolds

$$|\sigma^1_A|^2 + |\sigma^2_A|^2 - |\rho^1_A|^2 - |\rho^2_A|^2 - |\Phi^I_A|^2 = \frac{1}{2}$$
$$(\sigma^1_A)^2 + (\sigma^2_A)^2 - (\rho^1_A)^2 - (\rho^2_A)^2 - (\Phi^I_A)^2 = 0$$

$$\sigma^1_A = \frac{1 + T_A U_A - (Z^I_A)^2}{2Y_A^{1/2}}, \qquad \sigma^2_A = i\frac{T_A + U_A}{2Y_A^{1/2}}$$

$$\rho^1_A = \frac{1 - T_A U_A - (Z^I_A)^2}{2Y_A^{1/2}}, \qquad \rho^2_A = i\frac{T_A - U_A}{2Y_A^{1/2}}$$

$$\Phi^I_A = \frac{iZ^I_A}{2Y_A^{1/2}}, \qquad K_A = -log\ Y_A$$

The superpotential of the $N = 1$ supergravity is determined by the gravitino mass terms in $N = 4$ after the $Z^2 \times Z^2$ orbifold projections.

Gravitino mass term: $e^{K/2}\, W =$

$$(\phi_0 - \phi_1) f_{IJK} \Phi_1^I \Phi_2^J \Phi_3^I + (\phi_0 + \phi_1) \bar{f}_{IJK} \Phi_1^I \Phi_2^J \Phi_3^I$$

$$\Phi_A^I = \left\{ \sigma_A^1,\ \sigma_A^2;\ \rho_A^1,\ \rho_A^2,\ \Phi_A^I \right\}$$

Both f_{IJK} $\bar{f}_{IJK}$ are the gauge structure constants of the $N = 4$ "mother" theory.

In the heterotic, the term proportional to f_{IJK} give rise to a perturbative "electric gauging". The term proportional to $\bar{f}_{IJK}$ provide the non-perturbative "magnetic gauging".

• What is the origin of f_{IJK} $\bar{f}_{IJK}$ in the superstrings and M-theory?

• What are the deformation parameters of the 2d σ-model in correspondence with the $N = 4$ gauging coefficients f_{IJK} $\bar{f}_{IJK}$?

3. Fluxes and $N = 4$ Gauging

In general, the breaking of SUSY requires a gauging with non-zero f_{IJK} involving the fields

$\sigma^1_A,\ \sigma^2_A;\ \rho^1_A,\ \rho^2_A$ $\longrightarrow$ **gauging involving the $N = 4$ graviphotons**
$\longrightarrow$ **gauging of the R-symmetry**

In string and M-theory, f_{IJK} and $\bar{f}_{IJK}$ are generated by non-zero FLUXES:
Electric and Magnetic fluxes,
RR and fundamental p-form fields:

- **3-form fluxes H^3, in the NS-sector of heterotic, type IIA and type IIB**

- **F^p, p-form fluxes, in M-theory and in the RR sector of type IIA and type IIB**

• F^2 **2-form fluxes**, **in heterotic** ($E_8 \times E_8$ **or** $SO(32)$) **as well as in type I**

• ω^3 **3-form geometrical fluxes**, **in all strings and M-theory**

Special cases have already been studied:

• H^3 **in heterotic**

Derendinger, Ibanez, Nilles, 85, 86; Dine, Rohm, Seiberg, Witten, 85; Strominger, 86; Rohm, Witten, 86.

• **Simultaneous presence of NS, RR** H^3 **and** F^3 **in Type IIB.**

Frey, Polchinski, 02; Giddings, Kachru, Polchinski, 02; Kachru, Schulz, Trivedi, 03; Kachru, Schulz, Tripathy, Trivedi, 03; Derendinger, Kounnas, Petropoulos, Zwirner, 04.

• ω^3, H^3, F^2, **exact string solution via freely acting orbifold.**

$\longrightarrow$

Generalization of the Scherk–Schwarz deformation to superstring theory.

Rohm, 84;
Kounnas, Porrati, 88;
Ferrara, Kounnas, Porrati, Zwirner, 89;
Kounnas, Rostand, 90;
Kiritsis, Kounnas, 96;
Kiritsis, Kounnas, Petropoulos, Rizos, 99;
Antoniadis, Dudas, Sagnotti, 99;
Antoniadis, Derendinger, Kounnas, 99;
Derendinger, Kounnas, Petropoulos, Zwirner, 04, 05;
. . . .

4. Some examples of Geometrical Fluxes

• Breaking of supersymmetry a la Scherk-Schwarz

In the language of freely acting orbifolds, this corresponds to a twist induced by an R-symmetry operator and a shift in one internal coordinate.

The gravitino becomes massive due to the modification of the boundary conditions (in $D = 4$ Planck mass units)

$$m_{3/2}^2 = g^2 \ \frac{Q^2}{R^2}$$

Q is the R-symmetry charge
g_s is the string coupling constant
R is the compactification radius of the shifted coordinate.

What is the induced superpotential in the effective $N = 1$ description?

What is the flux interpretation of this specific model in the heterotic or type IIA orientifolds?

Choose the R-symmetry operator which induces the rotation in the ij plane

$$Q_{ij} = \oint dz \left[\Psi_i \Psi_j + x_i \partial x_j - (i \leftrightarrow j)\right]$$

Ψ_i $\rightarrow$ **2-d world sheet left-handed fermions**
x_i **the internal compactified coordinates.**

Strictly speaking, the operator Q is not well defined, since the internal coordinates are compactified $\rightarrow$ only discrete rotations are permitted $\leftrightarrow$ the crystallographic symmetries of the momentum lattice.

Switching on the deformation on the world sheet

$$\delta S_{ws} = F^{(k)}_{ij}\ Q_{ij}\ \bar{\partial} x_k\,,$$

corresponds to switch on a non-zero $F^{(k)}_{ij}$ $\rightarrow$ a magnetic flux of the graviphotons

$$A^{(k)}_M = G^k_M + B^k_M, \qquad M = i, j$$

G^k_M and B^k_M are the $D = 10$ metric and antisymmetric tensor fields compactified on a S^1 cycle associated with x^k.

Only discrete rotations make sense $\rightarrow$ quantization of the magnetic fluxes.

The structure constant coefficients f^K_{IJ} of the $N = 4$ gauged supergravity are given in terms of the magnetic fluxes $F^{(k)}_{ij}$.

The induced superpotential in the $N = 1$ language (after the $Z_2 \times Z_2$ projections) reads

$$W = e^{-K/2} \; F^1_{2,3} \; (\sigma^1_1 + \rho^1_1) \; \sigma^2_2 \; \sigma^2_3$$
$$= N_{flux} \; 1 \; (T_2 + U_2) \; (T_3 + U_3)$$

x^k **is taken in the 1st complex plane**
x^i **and** x^j **in the second and third planes**

Some comments are in order:

• **The shifted direction has to be taken left-right symmetric; that is the reason of the** $\sigma^l_1 + \rho^l_1$ **combination**

• **The choice of** $l = 1, 2$ **corresponds to the two directions of the 1st complex plane. The two choices are equivalent via** $U_1 \leftrightarrow 1/U_1$ **duality transformation**

• **The twisted directions are taken only left-moving. The R-symmetry operators in heterotic are left-moving. This is the reason that only the σ_i^l appear in the superpotential. Here also the choice of $l = 2$ is equivalent to the $l = 1$ by means of U_i-duality transformations**

Having the $N = 1$ superpotential and the Kähler potential

$$K = -\log(S+\bar{S}) - \sum_{A=1}^{3}[\log(T_A+\bar{T}_A)+\log(U_A+\bar{U}_A)]$$

we can determine the potential.

The potential is flat in the field directions S, T and U with broken supersymmetry. (no-scale model)

$$\frac{G_S G_{\bar{S}}}{G_{S\bar{S}}} = \frac{G_{T_1} G_{\bar{T}_1}}{G_{T_1\bar{T}_1}} = \frac{G_{U_1} G_{\bar{U}_1}}{G_{U_1\bar{U}_1}} = 1$$

$$\frac{V}{N^2} = \frac{|T_2 - \bar{U}_2|^2 |T_3 + U_3|^2 + |T_3 - \bar{U}_3|^2 |T_2 + U_2|^2}{2^6 ReS ReU_1 ReT_1 ReU_2 ReT_2 ReU_3 ReT_3}$$

$$T_A = \bar{U_A}, \quad A = 2,3 \qquad \text{at the minimum}$$

The gravitino mass is *independent* **of the moduli** $T_A, U_A, \ A = 2,3$

$$m^2_{3/2} = \frac{N^2}{(S + \bar{S})(U_1 + \bar{U}_1)(T_1 + \bar{T}_1)} = g_s^2 \ \frac{Q^2}{R_1^2}$$

- $SU(2)_k \times SU(2)_{k'}$ **- gauging in heterotic**

The $N = 1$ **superpotential is determined from the left- and right- moving structure constants of the left- and right-moving** $SU(2)_k \times SU(2)_{k'}$**. This generates non trivial** σ_A **and** ρ_A **terms in the superpotential**

$$W = \ e^{-K/2} \ A_l \ (\ \sigma_1^l \ \sigma_2^l \ \sigma_3^l + \rho_1^l \ \rho_2^l \ \rho_3^l \ )$$

$$W = iN\ (T_1+U_1)(T_2+U_2)(T_3+U_3)$$
$$+iN\ (T_1-U_1)(T_2-U_2)(T_3-U_3)$$
$$+N'\ (T_1U_1+1)(T_2U_2+1)(T_3U_3+1)$$
$$+N'\ (T_1U_1-1)(T_2U_2-1)(T_3U_3-1)$$

After minimization of the potential:

$$\frac{G_S G_{\bar S}}{G_{S\bar S}} = 1$$
$$\frac{G_{T_A} G_{\bar T_A}}{G_{T_A \bar T_A}} = \frac{G_{U_A} G_{\bar U_A}}{G_{U_A \bar U_A}} = 0, \qquad A = 1,2,3$$
$$T_A = \bar T_A = U_A = \bar U_A = 1 \qquad A = 1,2,3$$

The potential is negative with runaway behavior in the S direction

$$V = -2\ m_{3/2}^2 = -2\ \frac{N^2 + N'^2}{(S+\bar S)}$$

This is precisely the form of the Dilaton potential in the heterotic theory on $SU(2)_k \times SU(2)_{k'}$.

Indeed, because of the central charge deficit $\delta\hat{c}$ coming from the $SU(2)_k \times SU(2)_{k'}$ six - dimensional compactification

$$\delta\hat{c} = -\frac{4}{k+2} - \frac{4}{k'+2}$$

a negative potential is generated which in the Einstein frame takes precisely the above form with

$$N^2 = \frac{2}{k+2}, \qquad N'^2 = \frac{2}{k'+2}$$

- **$SU(2)_k \times SU(2)_{k'}$ perturbative and non-perturbative gauging in heterotic**

$$W = -iS\ W[SU(2)_k] + W[SU(2)_{k'}]$$

$$\begin{aligned} W = &\ S\ N\ (T_1 + U_1)(T_2 + U_2)(T_3 + U_3) \\ &+ S\ N\ (T_1 - U_1)(T_2 - U_2)(T_3 - U_3) \\ &+ N'\ (T_1 U_1 + 1)(T_2 U_2 + 1)(T_3 U_3 + 1) \\ &+ N'\ (T_1 U_1 - 1)(T_2 U_2 - 1)(T_3 U_3 - 1) \end{aligned}$$

After minimization of the potential:

$$\frac{G_S G_{\bar{S}}}{G_{S\bar{S}}} = \frac{G_{T_A} G_{\bar{T}_A}}{G_{T_A \bar{T}_A}} = \frac{G_{U_A} G_{\bar{U}_A}}{G_{U_A \bar{U}_A}} = 0, \qquad A = 1, 2, 3$$

$$S = \frac{N'}{N}, \quad T_A = \bar{T}_A = U_A = \bar{U}_A = 1, \quad A = 1, 2, 3$$

Stabilization of all moduli $\rightarrow$ **AdS$_4$-solution with unbroken supersymmetry**

$$V = -3m_{3/2}^2$$

This is similar to the stabilization of all the moduli found recently in Type IIA, D_6 **orientifold, by combining the RR-fluxes and the geometrical fluxes suitably.**

The $N = 4$ **gauging found in type IIA was based is based on** $SU(2)_k \times E^3_{k'}$

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5. Effective $N = 1$ superpotential from general fluxes

$\omega^3, \quad H^3, \quad H^2$ **In heterotic**

$\omega^3, \quad H^3, \quad H^2$ **In Type II asymmetric**

$\omega^3, \quad H^3, F^6, \quad F^4, \quad F^2, \quad F^0$ **in Type IIA**

$F^1, \quad F^3, \quad H^3, \quad \omega^3$ **in Type IIB**

Fluxes in the heterotic and IIB orientifolds are relatively well studied. IIA orientifolds have been explored to lower extent.

In the heterotic the complex fields

$$S, \ T_1, \ T_2, \ T_3, \ U_1, \ U_2, \ U_3$$

are defined in terms of the geometrical moduli G_{IJ} the dilaton Φ and B_{IJ}, $B_{\mu\nu} \sim a$

$$(G_{IJ})_A = \frac{t_A}{u_A}\begin{pmatrix} u_A^2+\nu_A^2 & \nu_A \\ \nu_A & 1 \end{pmatrix}$$

$$T_A = t_A + i\,(B_{IJ})_A\,, \quad U_A = u_A + i\nu_A$$

$$e^{-2\Phi_{10}} = s(t_1t_2t_3)^{-1}, \;\; S = s{+}ia, \quad g_{\mu\nu} = s^{-1}\tilde{g}_{\mu\nu}$$

The supersymmetric complexification in type IIA orientifolds is different, due to a dilaton rescaling and due to the orientifold projections.

$T'_A = T_A,$ As in the heterotic

$$s' = \sqrt{\frac{s}{u_1u_2u_3}}, \qquad u'_1 = \sqrt{\frac{su_2u_3}{u_1}},$$

$$u'_2 = \sqrt{\frac{su_1u_3}{u_2}}, \qquad u'_3 = \sqrt{\frac{su_1u_2}{u_3}}.$$

The imaginary components of S, U_A are given now by the 3-form fields

$$S = s' + i\ A_{6,8,10}, \qquad U_1 = u_1' + i\ A_{6,7,9}$$
$$U_2 = u_2' + i\ A_{5,8,9}, \qquad U_3 = u_3' + i\ A_{5,7,10}$$

- $N = 1$ **Superpotentials** $\leftrightarrow$ **IIA Fluxes**
- F^0 **- flux,** $\quad W = F_0$
- F^6 **- flux,** $\quad W = iF_6\ T_1T_2T_3$
- F^2 **- fluxes**

$$W(F^2) = F_{56}\ T_2T_3 + F_{78}\ T_3T_1 + F_{910}\ T_1T_2$$

- F^4 **- fluxes**

$$W(F^4) = iF_{78910}\ T_1 + iF_{56910}\ T_2 + iF_{5678}\ T_3$$

• H^3 - **fluxes**

$$W(H_3) = iH_{579}\ S + iH_{5810}\ U_1$$
$$+iH_{6710}\ U_2 + iH_{689}\ U_3$$

• ω^3 - **fluxes**

$$W(\omega_3) = \omega_{6810}\ T_1U_1 + \omega_{8106}\ T_2U_2 + \omega_{1068}\ T_3U_3$$
$$+\omega_{679}\ ST_1 + \omega_{895}\ ST_2 + \omega_{1057}\ ST_3$$

Type IIA Combined Fluxes, Gauging and Moduli Stabilization.

• Flat gaugings, no-scale models; stabilization of four moduli.

(i) Scherk–Schwarz, perturbative, ω^3-fluxes.

$$W = a\ (\ T_1U_1 + T_2U_2\ )$$

$V \geq 0$**, flat in** S, T_3, U_3 **directions**

$$m_{3/2}^2 = \frac{|a|^2}{s t_3 u_3}$$

(ii) Scherk–Schwarz non-perturbative,
ω^3, F^2**,** H^3, F^6**–fluxes**

$$W = a(\ ST_1 + T_2 T_3\ ) + ib(\ S + T_1 T_2 T_3\ )$$

$$m_{3/2}^2 = \frac{|2a|^2 + |2b|^2}{u_1 u_2 u_3}$$

(iii) $S0(3) \times S0(1,2)$**,** $E_3^c \times E_3^{nc}$ **gaugings**
ω^3, F^2**,** H^3, F^6**– fluxes**

$$W = a(\ ST_1 + ST_2 + ST_3\ ) + a(\ T_1 T_2 + T_2 T_3 + T_3 T_1\ )$$
$$+ i3b(\ S + T_1 T_2 T_3\ )$$

$$m_{3/2}^2 = \frac{|6a|^2 + |6b|^2}{u_1 u_2 u_3}$$

- **Non-compact gaugings,** $SO(1,2)$**,** E_3^{nc}
$V > 0$ **, runaway solutions.**

(i) One-modulus stabilization

$$W = F_0, \qquad W = iF_6\, T_1 T_2 T_3, \qquad W = iH_3\, S,$$
$$W = iH_3\, U_A, \qquad W = F_2\, T_A T_B, \qquad W = iF_4\, T_A, \ldots.$$

$$m_{3/2}^2 = \frac{|F_0|^2}{t_1 t_2 t_3 u_1 u_2 u_3}, \qquad V = 4 m_{3/2}^2$$

All others by dualities:

$T_A \to 1/T_A$, $U_A \to T_A$, $U_A \to S$

(ii) Two-moduli stabilization

$$W = F_0 + F_2\, T_1 T_2, \qquad W = iF_4\, (\, T_1 + T_2\,),$$
$$W = iH_3\, (\, S + U_1\,), \qquad W = iH_3\, (\, U_1 + U_2\,), \ldots$$

$$m_{3/2}^2 = \frac{|2F_0|^2}{s t_3 u_1 u_2 u_3}, \qquad V = 2 m_{3/2}^2$$

All others by dualities $U \leftrightarrow T \leftrightarrow S$

(iii) Three-moduli stabilization,
E_3^{nc} gauging, F_0, F_2, F_4, F_6–fluxes

$$W = a(1 + T_1T_2 + T_2T_3 + T_3T_1) + ib(T_1 + T_2 + T_3 + T_1T_2T_3)$$

$$m_{3/2}^2 = \frac{|4a|^2 + |4b|^2}{su_1u_2u_3}, \qquad V = m_{3/2}^2$$

- **Compact gaugings $SU(2)$, E_3^c**

(i) Stabilization of six-moduli,
NS_5 brane solution + linear dilaton

$$W = \omega_3(\ T_1U_1 + T_2U_2 + T_3U_3\ ) - F_0$$

$$V = -2m_{3/2}^2, \qquad m_{3/2}^2 = \frac{|2F_0|^2}{s}$$

(ii) Stabilization of all moduli, AdS_4-solution

Perturbative + Non-Perturbative $SU(2) \times E_3^c$ gauging.

$$W = iB\ [\ 2S + 5T_1T_2T_3 + 2(U_1 + U_2 + U_3) \\ -3(T_1 + T_2 + T_3)\ ] \\ +A\ [\ 2S(T_1 + T_2 + T_3) - (T_1T_2 + T_2T_3 + T_3T_1) \\ +6(T_1U_1 + T_2U_2 + T_3U_3) - 9\ ]$$

Both the EVEN term proportional to A and the ODD term proportional to B minimize to $S = T_A = U_A = 1$ with.

$$V = -3m_{3/2}^2$$

The gauging constraints and the antisymmetry of structure constants f_{IJK}, imply that both EVEN and ODD products of S, T_A, U_A fields have to coexist.

The gauging constraints involve the flux coefficients:

$$6A^2 = 10B^2 \qquad \to \qquad B = A\ b, \qquad b = \sqrt{3/5}.$$

To make them integer (as they should be), it is necessary to rescale them

$$S, T_A, U_A \longrightarrow b\ (\ S, T_A, U_A\ )$$

$\longrightarrow$ **the minimum will be at** $S = T_A = U_A = 1/b$**.**

After this rescaling the superpotential becomes:

$$W = iN\,[\ 2S + 3T_1T_2T_3 + 2(U_1 + U_2 + U_3)\ ]$$
$$-3iN(T_1 + T_2 + T_3)$$
$$+N\,[\ 2S(T_1 + T_2 + T_3) - (T_1T_2 + T_2T_3 + T_3T_1)\ ]$$
$$+6N\,[\ (T_1U_1 + T_2U_2 + T_3U_3) - 15\ ]$$

Conclusion

Illustration and application of a general method that relates the $N = 1$ effective Kähler potential and the superpotential to a consistent orbifold and/or orientifold projections of gauged $N = 4$ supergravity.

Derivation of the effective superpotential $N = 4 \longrightarrow N = 1$ for the main moduli in the presence of general fluxes.

We identify the correspondence between various admissible fluxes, $N = 4$ gauging and $N = 1$ superpotential terms.

Construction of explicit examples with different features:

- **Stabilization of four moduli, $V \geq 0$:** **No-scale models.**

- **Stabilization of less than four moduli, $V > 0$: de Sitter like, runaway solutions with possible cosmological interest.**

- **Models based on compact "gaugings", $V < 0$: Domain-Wall Solutions, Five-brane solutions with non trivial Dilaton or else.**

- **Models which admit a supersymmetric AdS_4 vacuum with all moduli stabilized.**