A model for the present universe as a global attractor

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Outline

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Introduction

-According to observational data the Universe today is accelerating

- Within the framework of GR the second Friedmann equation:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(1+3w)\rho$$

-Divide fluids into 2 categories:

a) Decelerating ($w > -1/3$), b) Accelerating ($w < -1/3$)

- Simpler candidate: Cosmological constant (CC) (like a perfect fluid with $w = -1 =$ constant)

- Other candidates: Scalar fields coupled to gravity (e.g. phantom, quintessence, with a varying w) with some self-interaction potential

In general, dark energy models are divided into two broad categories:

-Dynamical dark energy, in which one modifies the matter side of the field equations

$$G_{\mu\nu} = 8\pi G(T_{\mu\nu} + T_{\mu\nu}^{dark})$$

-Geometrical dark energy, in which one modifies the geometric side of the field equations

$$(G_{\mu\nu} + G_{\mu\nu}^{dark}) = 8\pi G T_{\mu\nu}$$

- A simple modified-gravity model is the DGP brane-world model (as generalized to cosmology by Deffayet)
- We are confined on a 3d hypersurface, called the brane, embedded in a higher-dimensional spacetime and gravity resides in the whole bulk
- Brane-worlds are inspired from String/M-theory and contain the basic ingredients (extra dimensions, extended objects etc). They include corrections to usual Einstein-Hilbert term for gravity
- A IR correction is the induced gravity (IG) term on the brane (important at low energies)
- A UV correction is the Gauss-Bonnet (GB) term in the bulk (important at high energies, e.g. inflation)
- Interested in cosmic acceleration $\rightarrow$ IG correction

The model

Action for the model

$$S = \int d^5x\sqrt{-g}\,(M^3 R - \Lambda) + \int d^4x\sqrt{-h}\,(m^2\hat{R} - V)$$

plus matter in the bulk and perfect fluid on the brane

Einstein's equations

$$G_{AC} = \frac{1}{2M^3} T_{AC}|_{tot}$$

where

$$T^A_C|_{tot} = T^A_C|_{v,B} + T^A_C|_{m,B} + T^A_C|_{v,b} + T^A_C|_{m,b} + T^A_C|_{ind}$$

Seek for cosmological solutions

$$ds^2 = -n(t,y)^2 dt^2 + a(t,y)^2 \gamma_{ij} dx^i dx^j + b(t,y)^2 dy^2$$

Compute the Einstein tensor

$$G_{00}=3\left\{\frac{\dot a}{a}\left(\frac{\dot a}{a}+\frac{\dot b}{b}\right)-\frac{n^2}{b^2}\left[\frac{a''}{a}+\frac{a'}{a}\left(\frac{a'}{a}-\frac{b'}{b}\right)\right]+\frac{kn^2}{a^2}\right\}$$

$$G_{ij}=\frac{a^2}{b^2}\gamma_{ij}\left\{\frac{a'}{a}\left(\frac{a'}{a}+\frac{2n'}{n}\right)-\frac{b'}{b}\left(\frac{n'}{n}+\frac{2a'}{a}\right)+\frac{2a''}{a}+\frac{n''}{n}\right\}$$
$$+\frac{a^2}{n^2}\gamma_{ij}\left\{\frac{\dot a}{a}\left(\frac{2\dot n}{n}-\frac{\dot a}{a}\right)-\frac{2\ddot a}{a}+\frac{\dot b}{b}\left(\frac{\dot n}{n}-\frac{2\dot a}{a}\right)-\frac{\ddot b}{b}\right\}-k\gamma_{ij}$$

$$G_{05}=3\left(\frac{n'}{n}\frac{\dot a}{a}+\frac{a'}{a}\frac{\dot b}{b}-\frac{\dot a'}{a}\right)$$

$$G_{55}=3\left\{\frac{a'}{a}\left(\frac{a'}{a}+\frac{n'}{n}\right)-\frac{b^2}{n^2}\left[\frac{\ddot a}{a}+\frac{\dot a}{a}\left(\frac{\dot a}{a}-\frac{\dot n}{n}\right)\right]-\frac{kb^2}{a^2}\right\}$$

Compute the various energy-momentum tensors

$$T^A_C|_{v,B} = \mathrm{diag}(-\Lambda, -\Lambda, -\Lambda, -\Lambda, -\Lambda)$$
$$T^A_C|_{v,b} = \mathrm{diag}(-V, -V, -V, -V, 0)\frac{\delta(y)}{b}$$
$$T^A_C|_{m,b} = \mathrm{diag}(-\rho, p, p, p, 0)\frac{\delta(y)}{b}$$

$$T^0_0|_{ind} = \frac{6m^2}{n^2}\left(\frac{\dot{a}^2}{a^2} + \frac{kn^2}{a^2}\right)\frac{\delta(y)}{b}$$
$$T^i_j|_{ind} = \frac{2m^2}{n^2}\left(\frac{\dot{a}^2}{a^2} - \frac{2\dot{a}\dot{n}}{an} + \frac{2\ddot{a}}{a} + \frac{kn^2}{a^2}\right)\delta^i_j\frac{\delta(y)}{b}$$

Define new quantities

$$
\begin{aligned}
r_c &= \frac{m^2}{M^3} \\
\lambda &= \frac{2V}{m^2} + \frac{12}{r_c^2} - \frac{\Lambda}{M^3} \\
\mu &= \frac{V}{6m^2} + \frac{2}{r_c^2} \\
\gamma &= \frac{1}{12m^2} \\
\beta &= \frac{1}{\sqrt{3}r_c}
\end{aligned}
$$

Derive the cosmological equations on the brane

$$\dot{\rho} + 3H(1+w)\rho = -T$$

$$H^2 = \mu + 2\gamma\rho + \beta\psi - \frac{k}{a^2}$$
$$\dot{\psi} + 2H\left(\psi - \frac{\lambda + 6(1-3w)\gamma\rho}{\psi}\right) = \frac{2\gamma T}{\beta}$$
$$\frac{\ddot{a}}{a} = \mu - (1+3w)\gamma\rho + \beta\frac{\lambda + 6(1-3w)\gamma\rho}{\psi}$$

where $\psi \equiv \sqrt{\lambda + 24\gamma\rho + \chi}$ and assume $T(\rho) = A\rho^\nu$, $\nu > 0$

Analysis

Define new variables (for flat universe, $k = 0$)

$$\omega_m = \frac{2\gamma\rho}{D^2} \quad , \quad \omega_\psi = \frac{\beta\psi}{D^2} \quad , \quad Z = \frac{H}{D}$$

New equations

$$\omega_m + \omega_\psi = 1$$

$$\omega_m' = \omega_m \Bigg[(1+3w)(\omega_m - 1)Z - \frac{A}{\sqrt{|\mu|}} \left(\frac{|\mu|\omega_m}{2\gamma} \right)^{\nu-1} (1-Z^2)^{\frac{3}{2}-\nu} - 2Z(1-Z^2)\frac{1-Z^2-3(1-3w)\beta^2\mu^{-1}\omega_m}{1-\omega_m} \Bigg]$$

$$Z' = (1-Z^2)\Bigg[(1-Z^2)\frac{1-Z^2-3(1-3w)\beta^2\mu^{-1}\omega_m}{1-\omega_m} - 1 - \frac{1+3w}{2}\omega_m \Bigg]$$

1. Fixed points:

- Fixed point: If initial condition, the system stays there. It is found by setting derivatives equal to 0
- Character: Attractor, repeller, saddle (depends on the characteristic matrix)
- Attractor: 2 eigenvalues (real part) negative

Repeller: 2 eigenvalues (real part) positive

Saddle: 1 eigenvalue positive, 1 negative

The results are summarized in the tables

	$\nu < 3/2$	$\nu = 3/2$	$\nu > 3/2$
No. of F.P.	1	0 or 1	1
Nature	A	A	S

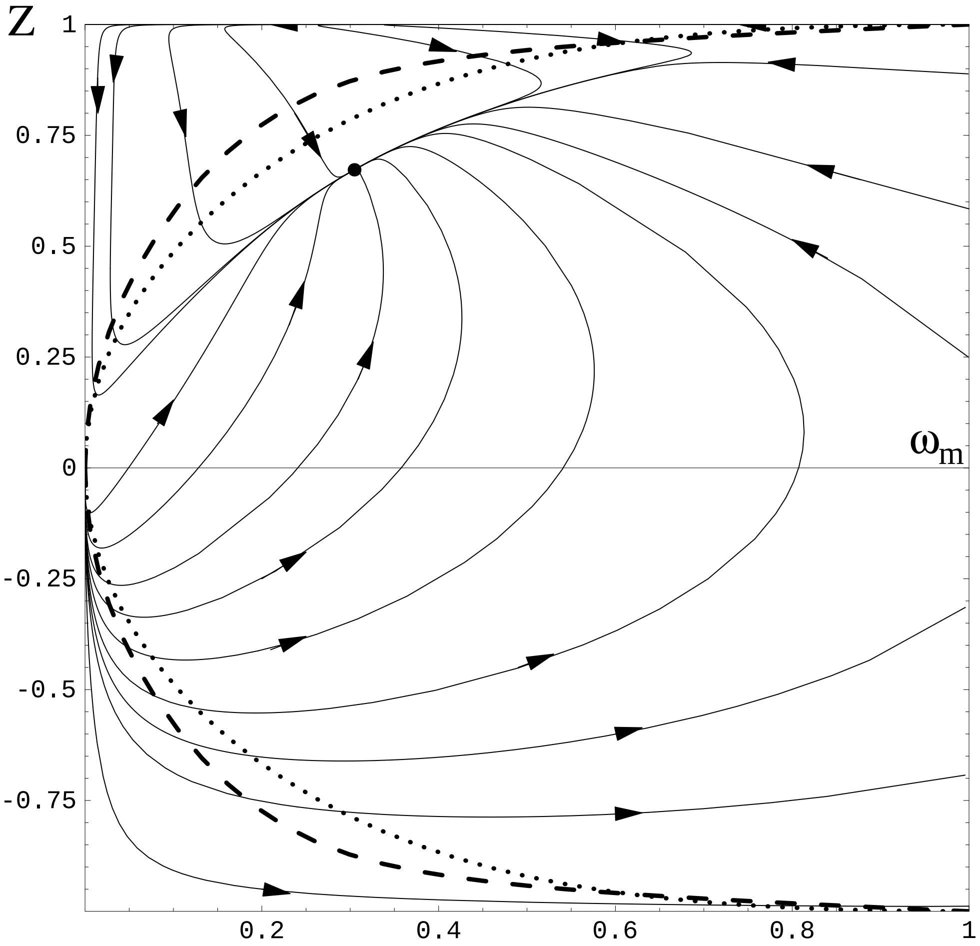
Table 1: The fixed points for w=0, influx

	$\nu < 3/2$	$\nu = 3/2$	$3/2 < \nu < 2$	$\nu = 2$	$\nu > 2$
No. of F.P.	1	0 or 1	0 or 2	0 or 1	1
Nature	A	A	A,S	S	S

Table 2: The fixed points for w=1/3, influx

2. Phase-portraits ($\omega_m - Z$ plane)

The history of the universe can be shown



Influx, $w = 0$, $\nu < 3/2$

3. Dark energy: It is a result of the geometry and the energy exchange

We can write a semi-conservation law for dark energy

$$\dot{\rho}_{DE} + 3(1+w_{DE})H\rho_{DE} = T$$

where w_{DE} is

$$w_{DE} = \frac{-1}{3(1-\omega_m)}\left[2Z^2 - \omega_m - 1 - 6(1-3w)\frac{\beta^2}{\mu}\frac{\omega_m(1-Z^2)}{Z^2-\omega_m}\right]$$

Evolving in time

At the fixed point we obtain

$$w_{DE*} = -1 - \frac{1+w}{\Omega_{m*}^{-1} - 1}$$

and for $\Omega_{m*} = 0.3$, $w_{DE*} = -1.4$

Conclusions

- In our scenario, the present universe can be easily realized as a unique attractor of the dynamical system

- The dark energy is a result of the geometry and the energy exchange

- The parameter w of the dark energy evolves in time and crosses the $w = -1$ barrier from larger to lower values

- For $\Omega_M = 0.3$, the parameter w of the dark energy is today $w_0 = -1.4$, independently of the parameters of the models