Evolution of Critical Corelations in the QCD Phase Transition

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Framework

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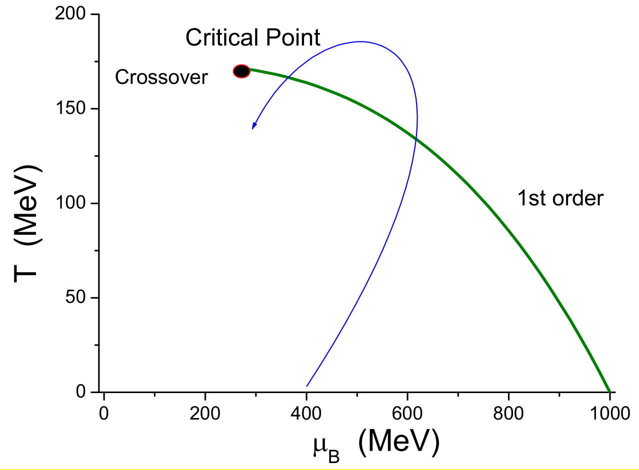
- It is believed that in the QCD phase diagram (T, μ) there exists a 1st order phase transition line above which the system lies in the chirally symmetric phase. This line ends at a critical point, where the phase transition becomes 2nd order, and beyond which it is replaced by analytical crossovers.



Goal

- In a heavy-ion collision experiment, the fireball is believed to achieve a chirally symmetric phase and subsequently return to the ordinary hadronic phase crossing the 1st order phase transition line as it freezes out.
- If during this transition the system reaches chemical and thermal equilibrium in the neighborhood of the critical point (T_c, μ_c), it will acquire critical characteristics such as critical correlations.

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- If during this transition the system reaches chemical and thermal equilibrium in the neighborhood of the critical point (T_c, μ_c), it will acquire critical characteristics such as critical correlations.

- The critical state and the corresponding correlations have a finite life-time due to the dynamics.

- Our goal is to study the evolution of these correlations and the possibility to leave signals at the detectors.

The Model

As an effective description of the chiral theory we use the σ-model [Rajagopal and Wilczek, 1993]. The 3-dimensional Lagrangian density is

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \vec{\pi} \partial^\mu \vec{\pi}) - V(\sigma, \vec{\pi}),$$

with the potential

$$V(\sigma, \vec{\pi}) = \frac{\lambda^2}{4}(\sigma^2 + \vec{\pi}^2 - v_0^2)^2 + \frac{m_\pi^2}{2}\left(\sigma^2 + \vec{\pi}^2 - 2v_0\sigma + v_0^2\right)$$

where $\sigma = \sigma(\vec{x}, t)$ and $\vec{\pi} = \vec{\pi}(\vec{x}, t)$. The scalar field σ together with the pseudoscalar field $\vec{\pi} = (\pi^+, \pi_0, \pi^-)$ form a chiral field $\Phi = (\sigma, \vec{\pi})$.

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- The second term in the potential accounts for the explicit chiral symmetry breaking by the quark masses. We use the phenomenological values $m_\pi \approx 139$ MeV, $v_0 \approx 87.4$ MeV, and $\lambda^2 \approx 20$.

Equations of Motion

- The equations of motion are

$$\ddot{\sigma} - \nabla^2\sigma + \lambda^2(\sigma^2 + \vec{\pi}^2 - v_0^2)\sigma + m_\pi^2\sigma = v_0 m_\pi^2$$
$$\ddot{\vec{\pi}} - \nabla^2\vec{\pi} + \lambda^2(\sigma^2 + \vec{\pi}^2 - v_0^2)\vec{\pi} + m_\pi^2\vec{\pi} = 0,$$

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- For initial conditions we use an ensemble of critical configurations for the σ-field and an ensemble of π-field configurations corresponding to an ideal gas at temperature T_0.

Initial Conditions, σ

- The partition function of the σ-field in thermal equilibrium is given by: $Z = \int \delta[\sigma] e^{-\Gamma[\sigma]}$ where the free energy near the critical point is:

$$\Gamma[\sigma] = \int_V d^D x \{ \frac{1}{2} (\nabla \sigma)^2 + g \sigma^{\delta+1} \}.$$

$D = 3$ is the dimensionality, $\delta = 5$ is the isothermal critical exponent, and the coupling $g = 2$, in order to describe the effective action of the $3d$ Ising model at its critical point [M. Tsypin (1994)].

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- The critical system is simulated producing σ-configurations distributed according the weight $e^{-\Gamma[\sigma]}$, through random partitioning of the lattice in elementary clusters of different volume V and a random choice for the constant value of the σ-field within each cluster. The σ ensemble is then formed by recording a large number of statistically independent σ-configurations, and their initial time derivative is assumed to be zero since we are in equilibrium.

Initial Conditions, σ

- So we can produce an ensemble of σ-configurations possessing critical fluctuations and this power-law behavior is depicted in $\langle \int_R |\sigma(\vec{x}-\vec{x}_0)\sigma(\vec{x}_0)|\, d^D x\rangle$, averaged inside clusters of volume V, with R the distance around a point $\vec{x}_0$, which is proportional to $R^{15/6}$.

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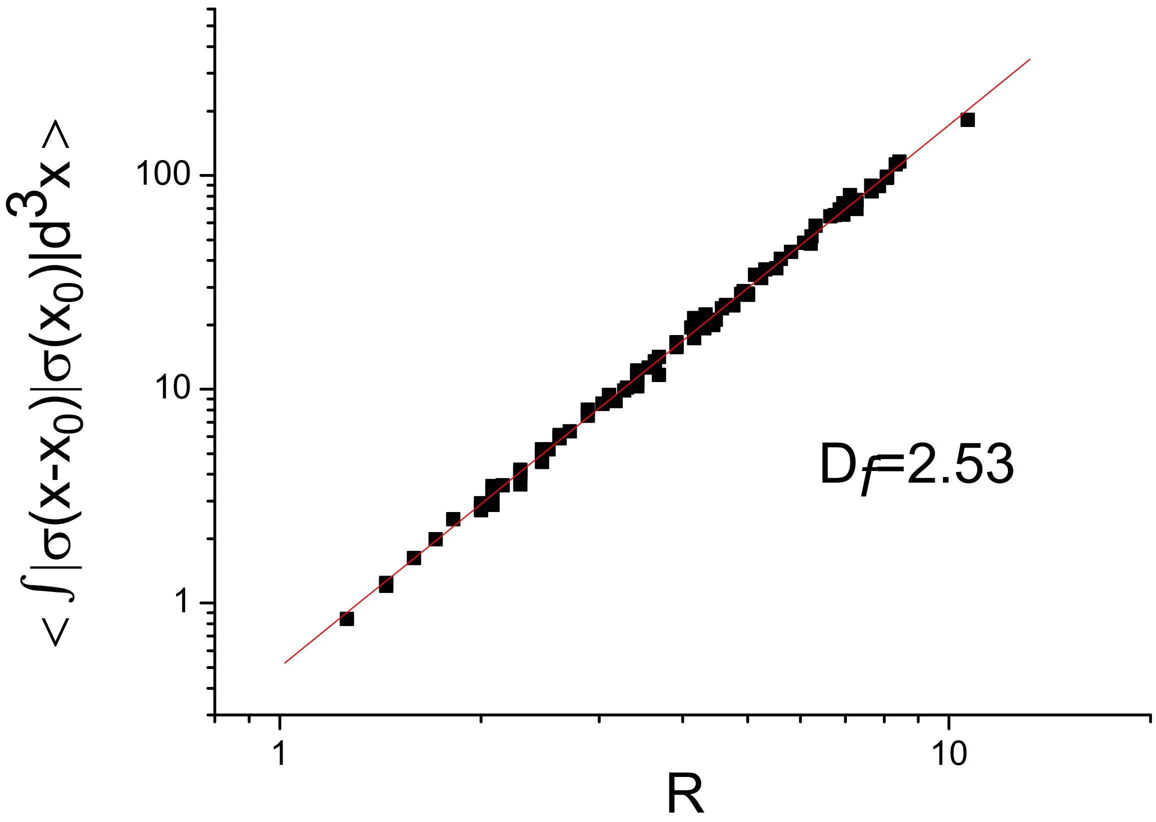
- $$\langle \int_R |\sigma(\vec{x}-\vec{x}_0)\sigma(\vec{x}_0)|\, d^D x\rangle \sim R^{D_F}$$

  for a system with fractal mass dimension $D_F = \frac{D\delta}{\delta+1}$ (where $D=3$, $\delta=5$). This measure is experimentally accessible since it is related to the density-density correlation of the σ-particles [N.G. Antoniou *et. al.* (2002)].

Initial Conditions, σ



Results will be sent in Comput. Phys.

Initial Conditions, π

We generalize [F. Cooper *et. al.* (2001)] in order to produce an ensemble of 3-d π-configurations corresponding to an ideal gas at temperature T_0.

The unperturbed Hamiltonian for the classical scalar field theory in 3-d is $H = \frac{1}{2}\int d^3x[(\partial_t \pi(\vec{x},t))^2 + (\nabla\pi(\vec{x},t))^2 + m_\pi^2 \pi(\vec{x},t)^2]$.

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- The free particle solutions for $t = 0$ are

$$\pi(\vec{x},0) = \int_{-\infty}^{+\infty} \frac{d^3k}{(2\pi)^3} \frac{(a_k + a^*_{-k})}{\sqrt{2\omega_k}} e^{i\vec{k}\vec{x}}$$
$$\dot{\pi}(\vec{x},0) = \int_{-\infty}^{+\infty} \frac{d^3k}{(2\pi)^3} \sqrt{\frac{\omega_k}{2}} i(a^*_{-k} - a_k) e^{i\vec{k}\vec{x}}. \qquad (2)$$

where $\omega_k = \sqrt{k^2 + m_\pi^2}$.

Initial Conditions, π

Now, choosing an initial classical density distribution [Cooper *et. al.* (2001)] $\rho[\pi,\dot{\pi}] = Z^{-1}(\beta_0)\,\exp\{-\beta_0\,H[\pi,\dot{\pi}]\}$, and substitute the Hamiltonian with the free particle solutions, we finally acquire:

$$\rho[x_k, y_k] = Z^{-1}(\beta_0)\,\exp\left\{-\beta_0\int_{-\infty}^{+\infty}\frac{d^3k}{(2\pi)^3}\omega_k(x_k^2+y_k^2)\right\}, \qquad (3)$$

with $\beta_0 = 1/T_0$, and we have split the complex a_k as $a_k = x_k + iy_k$ with x_k, y_k real.

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with $\beta_0 = 1/T_0$, and we have split the complex a_k as $a_k = x_k + iy_k$ with x_k, y_k real.

So if we want to produce a thermal ensemble (at temperature T_0) of configurations for $\pi(\vec{x},0)$ and $\dot{\pi}(\vec{x},0)$, we select x_k and y_k from the gaussian distribution (4), assemble a_k and then substitute in (2).
We independently repeat this procedure three times, since we have three components of the pion pseudoscalar field.

Initial Conditions, π

The π-correlator has the characteristic (for an ideal thermal gas) form of a δ-function, since there are no specific correlations and the only significant pattern comes for $\delta\vec{x}$=0 since in this case $C_\pi(\delta\vec{x}) = \langle\pi(\vec{x})\pi(\vec{x}+\delta\vec{x})\rangle - \langle\pi(\vec{x})\rangle\langle\pi(\vec{x}+\delta\vec{x})\rangle$ gives the standard deviation for each lattice site.

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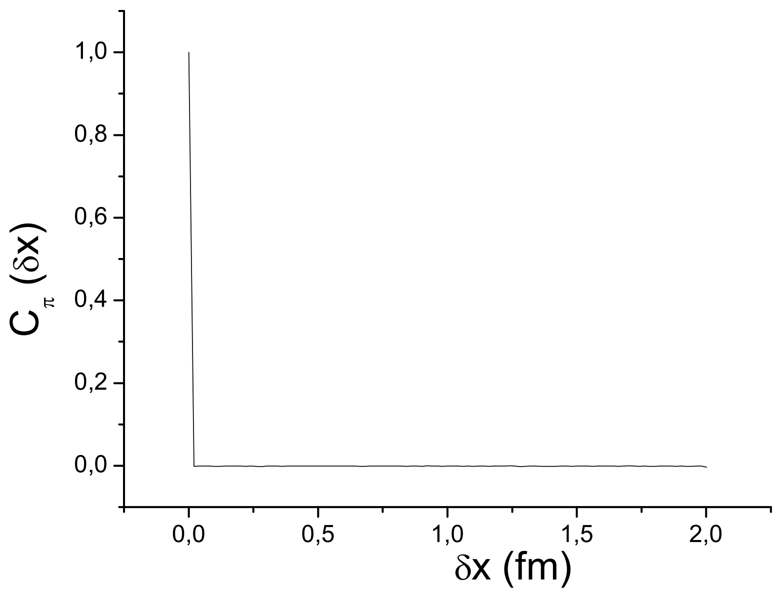

Numerical Results

- We solve the equations of motion in 3-d $20 \times 20 \times 20$ lattice, using for initial conditions an ensemble of 10^4 σ and π configurations satisfying the aforementioned requirements, that is critical σ-configurations and thermal π ones (at $T_0 \approx 140$ MeV).

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- We are interested in investigating the evolution of the $\langle \int_R |\sigma(\vec{x} - \vec{x}_0)\sigma(\vec{x}_0)| \, d^3x \rangle \sim R^{D_F}$ which initially has the characteristic power-law pattern with exponent $\psi(0) = D_f =$15/6.

Finite Time Quench

- Finally, in order to acquire the correct $m_\sigma \approx 0$ at $t = 0$ ($O(4)$ critical point), instead of $m_\sigma = \sqrt{2\lambda^2 v_0^2 + m_\pi^2}$, we assume a more physical, finite time, mechanism for chiral symmetry breaking, instead of the instant quench:

$$v(t) = v_0 t/\tau \qquad \text{for} \qquad t \leq \tau$$

$$v(t) = v_0 \qquad \text{for} \qquad t > \tau, \tag{5}$$

where τ is the quench duration and $v_0 \approx 87.4$ MeV is the zero temperature value of the potential minimum.

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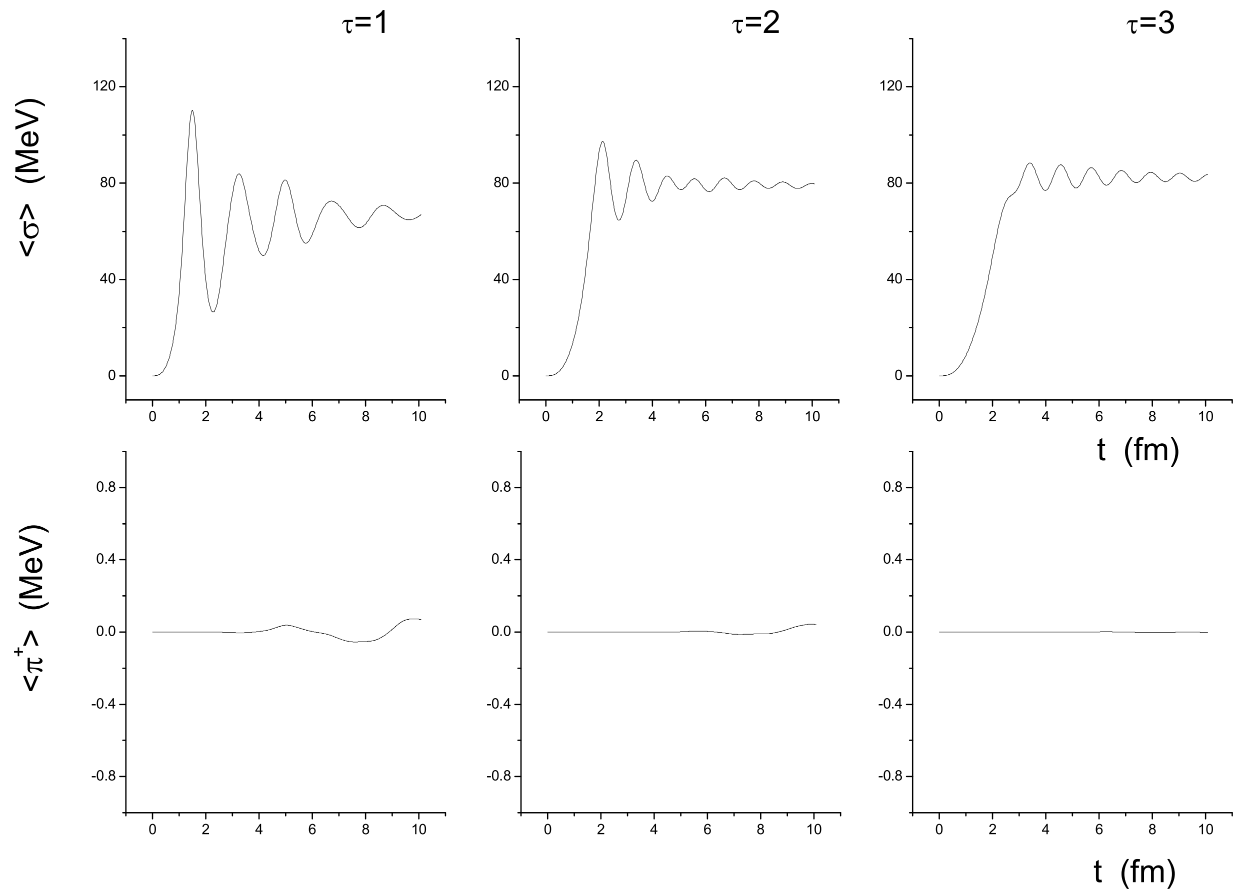

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- For the same cases we investigate the evolution of the exponent $\psi(t)$ of $\langle \int_R |\sigma(\vec{x} - \vec{x}_0)\sigma(\vec{x}_0)| \, d^3x \rangle$ vs R, which initially has the value of $\psi(0) = D_f = 15/6$.

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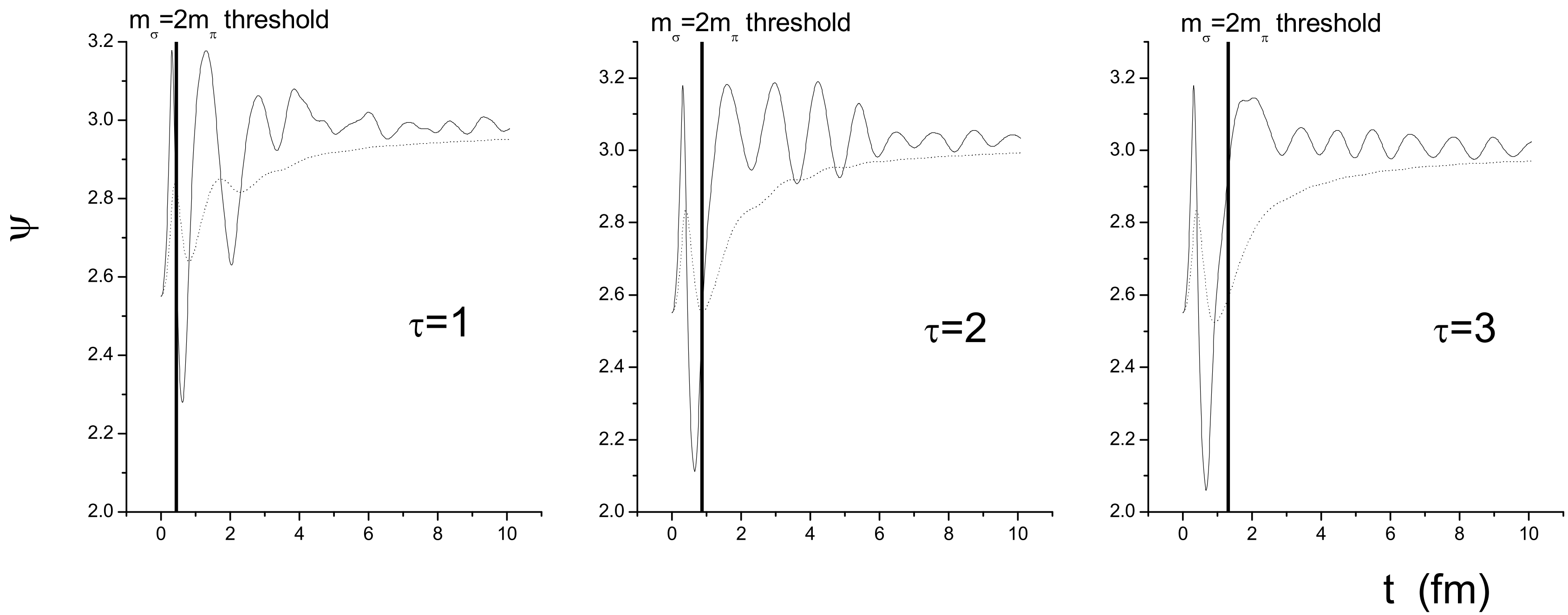

Numerical Results

- As we observe ψ and especially its time average $\langle\psi\rangle_t$ vary between 2.6 and 2.9, preserving slight traces of the initial power law of 15/6, but asymptotically reach to the embedded dimension value 3.
(Results have been submitted for publication to Phys. Rev. E)

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- That is, m_σ reaches the threshold of $2m_\pi$ during the freeze out process, in times where there is still a power law pattern in the σ-field.

- So through the decay of σ ($\sigma \to 2\pi$), there is a possibility to produce pions possessing signals of the σ critical fluctuations, and these pions can be detected.

Numerical Results

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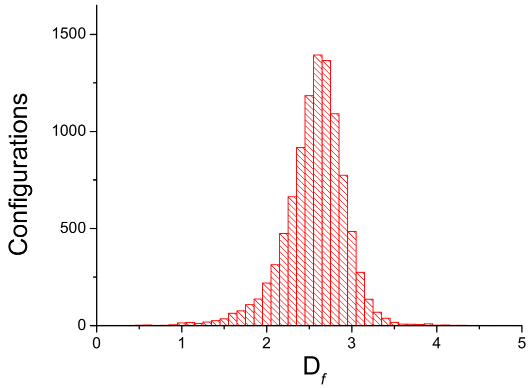
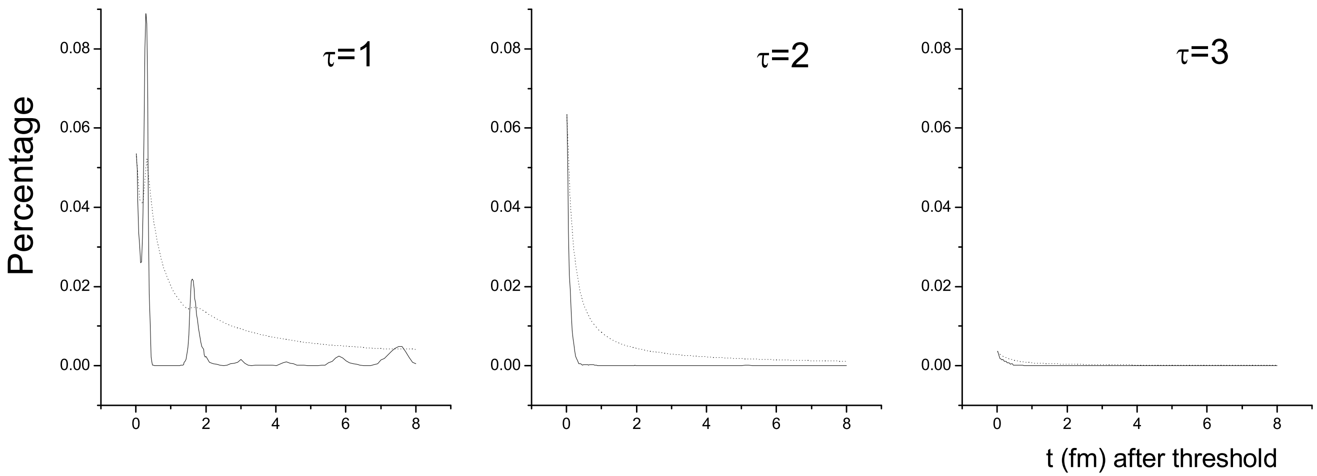
- We use an ensemble of σ-field configurations, each one possessing its own D_f, leading to a distribution with mean value $\approx 15/6$.



An event-by-event analysis consists in calculating the percentage of the initial configurations that have $D_f \approx 15/6$ in the beginning and possess again this value after the threshold $m_\sigma = 2m_\pi$.

Numerical Results



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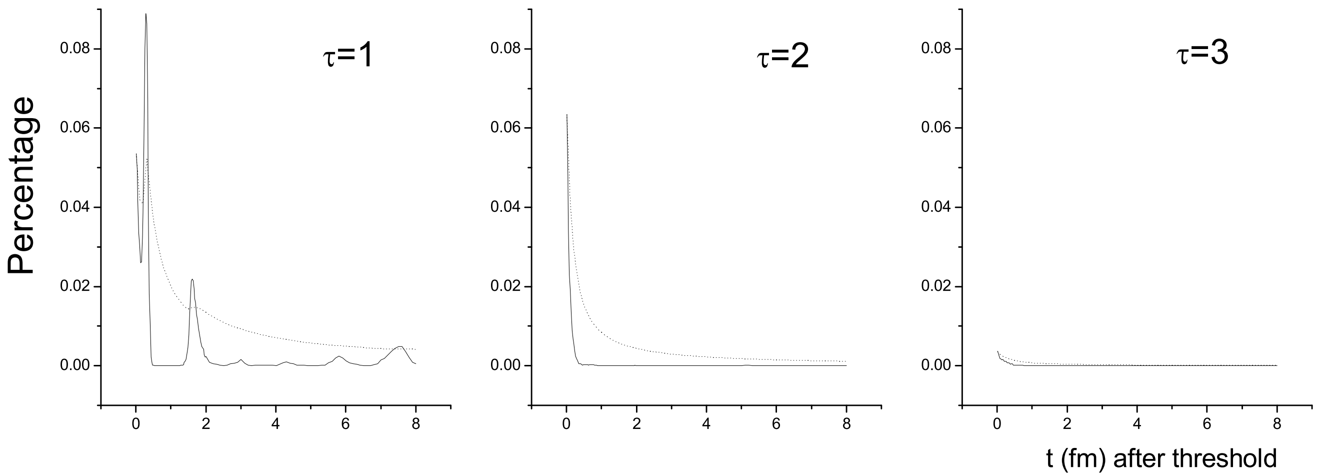

- This percentage is quite large, for quench times $\tau = 1$ and $\tau = 2$, and it stays over 0.5% almost for 6-7 fms after the $m_\sigma = 2m_\pi$ threshold.
- The time of the time average is not known exactly. However, it cannot be more than some fms since that is the time when m_σ reaches to its zero temperature value, where the decay rate becomes huge.

Numerical Results

- So there is a "cross section" $\geq 0.5\%$ for the σ's to transfer their initial critical profile through their decay to the produced pions.
- To exclude the inverse possibility, (events with critical geometry arising from conventional initial conditions), we evolve our system using an ensemble of σ-configurations with random initial conditions.

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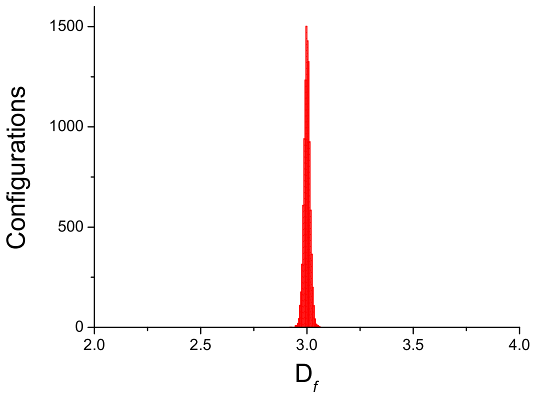

- This conventional profile always possesses $D_f \approx 3$ with a very small width. Not even one configuration acquire slope $\lesssim 2.96$ ever.
(Results will be sent to Phys. Rev. D)

Conclusions

- These results can be applied as a model for heavy ion collisions, if the system passes near the critical point, where we expect to reach equilibrium, σ to acquire the critical behavior mentioned above, and the pions to be thermal.

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- In this case the produced pions through $\sigma \rightarrow 2\pi$ decay may acquire critical characteristics, which can be detected, offering an experimental signal of the critical point.
- These results are valid for a variety of quench durations, and the "cross section" $\geq 0.5\%$ in the event-by-event analysis holds for sufficiently large times in order to reach to the freeze out.